Building Concepts: Building Expressions in Two Variables

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students gain more practice creating equivalent expressions using the properties of multiplication and addition. Students are able to build more complex expressions for the purpose of demonstrating expressions of the second and third degree.

Proper use of the properties of multiplication and addition helps to identify and create equivalent expressions involving rational numbers and explain why they are equivalent.

Learning Goals

1. Identify equivalent expressions involving rational numbers and explain why they are equivalent using properties of multiplication and addition;
2. understand that a counterexample can disprove a claim, but examples cannot establish a "proof";
3. identify patterns in a table and connect these patterns to the relationship between two expressions.

Prerequisite Knowledge

Building Expressions in Two Variables is the tenth lesson in a series of lessons that explore the concepts of expressions and equations. In this lesson students create equivalent expressions using the properties of multiplication and addition. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *Building Expressions* and *Equations and Operations*. Students should understand:

- the associative and commutative properties of addition and multiplication;
- how to interpret and write an expression;
- the concept of distributive property;
- the rules for operating with negative numbers.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **equivalent expressions:** two expressions that name the same number regardless of which value is substituted for the variables
- **rational numbers:** real numbers that can be written as a fraction
- **integers:** numbers that can be written without a fractional part
- **distributive property of multiplication over addition:** states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products together.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Building Expressions in Two Variables_Student.pdf
- Building Expressions in Two Variables_Student.doc
- Building Expressions in Two Variables.tns
- Building Expressions in Two Variables_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In Lesson 3 *Building Expressions*, students encountered linear expressions with one variable and used conventions about the order of operations and properties of operations to transform simple expressions into equivalent expressions. This lesson builds from this earlier work, introducing expressions that require understanding of the rules for operating with negative numbers and may involve two variables with rational coefficients and more operations. Experience with negative numbers allows students to view $7-2x$ as $7+(-2x)$ and to bring this perspective to thinking about equivalent expressions.

Students continue to work with equivalent expressions—two expressions that name the same number regardless of which value is substituted for the variables, i.e., produce the same output for all possible values of the variables in the two expressions. As students gain experience with multiple ways of writing an expression, they recognize that writing an expression in different ways can reveal a different way of seeing a problem. For example $2(x+3)$ indicates double the sum of *x* and 3, while the equivalent expression $2x+6$ indicates the sum of double *x* and 6.

Note that the TNS activity can be used to serve two different levels of learning, which for most students should come in distinct and separate parts of the learning trajectory. The first level involves relatively simple linear expressions where the goal is to prepare students to be flexible using negative integers in algebraic expressions of the form $a(x+b)+c(x+d)$ where *a*, *b*, *c*, and *d* are rational numbers. The second involves more complicated expressions of the second and third degree, where the goal is to prepare students for expanding and factoring polynomials. Care should be taken not to overreach with the first level.

Note that the TNS activity uses “opposite” for situations like $+-b$, for adding the opposite of *b*, rather than “add negative *b*”, reserving the word *negative* for actual numerical values.

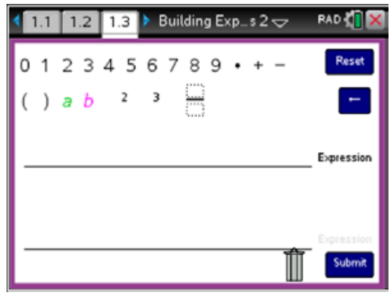
Building Concepts: Building Expressions in Two Variables

Part 1, Page 1.3

Focus: Create equivalent expressions and explain why they are equivalent.

On page 1.3, grab and drag symbols or use the keypad to create an expression on each line.

Submit displays the expression with two number lines.



To move the dots on the number lines, drag or use the right/ left arrows.

Edit returns to entry page.

Reset returns blank lines.

TI-Nspire Technology Tips

[menu] accesses page options.

[tab] cycles through the shapes or the blanks.

Up/Down arrow keys move between the numerator and denominator.

[÷] inserts the fraction.

[^] moves the cursor to the exponent.

[enter] submits the expressions and returns to edit.

[ctrl] [del] resets the page.

Class Discussion

Consider the expression 6b.

- ***Write out what 6b means in terms of addition.***

  Answer: $b+b+b+b+b+b$

- ***What do you think $6b-b$ means?***

  Answers may vary. For example, $b+b+b+b+b+b-b$ or $b+b+b+b+b$ or 6.

- ***On page 1.3, enter $6b-b$ in the first entry line and your answer to the question above in the second entry line and Submit. Describe what happens when you drag the point labeled b on the lower number line.***

  Answers may vary. The values of both expressions may be the same for all values of *b* or different depending on the answer to the question above.

Class Discussion (continued)

- ***Remember that two expressions are equivalent if they name the same number regardless of the value substituted for the variables. If your answer to the second question above did not produce equivalent expressions, select Edit and try another expression.***

Answers may vary. An equivalent expression could be 5*b* or $b+b+b+b+b$.

Consider the expression $6b+-b$. Remember from your work in Building Expressions that this is read as 6b plus the opposite of b.

- ***How does this expression differ from the expression in the question above?***

Answer: The expression $6b-b$ subtracts a *b* from 6*b*. The expression $6b+-b$ adds the opposite of *b* to 6*b*.

- ***Use the TNS activity to decide whether $6b+-b$ is equivalent to $6b-b$ and give a reason for your answer.***

Answers may vary. The two expressions are equivalent; justifications might include: "6*b* is the sum of six *b*'s. Adding one of these *b*s to the opposite of one *b* gives 0, leaving five *b*s." You get the same result if you take one *b* away from the 6 *b*s you have. Note that just showing they produce the same values as you change the variable is only an indication they are equivalent; justification has to come from the definition (or properties of an operation if appropriate).

- ***Serena claims $6b+b$ is equivalent to $6b^2$. What would you say to Serena? Give an example to help her thinking.***

Answer: Serena is not correct. The TNS activity gives different values for each expression as *b* changes because b^2 means the product of the two *bs,* not the sum. 7*b* or $4b+2b+b$ could be an equivalent expression.

Use the TNS activity to decide whether you agree or disagree with each of the following. Explain your thinking in each case.

- ***Pietro thought that $6b-b$ was equivalent to $7b-2b$.***

Answer: Pietro is correct because if you have 7 *b*s and take 2 *b*s away, you are left with five *b*s or 5*b* which is equivalent to the expression $6b-b$.

- ***Lien stated that $6b-b$ is equivalent to $2b+3b$.***

Answer: Lien is correct because both expressions are equivalent to 5*b.*

Class Discussion (continued)

- ***Thor thought $6b - b$ would be equivalent to $(6b+5)-(b+5)$. You just shift both the 6b and the b by five, so the difference will still be the same.***

Answer: Thor is correct; both expressions would be equivalent to 5*b.*

- ***Rei said that $12b - 2b$ would be equivalent to $6b - b$ because you doubled both of the parts, so the difference would be the same.***

Answer: Rei is not correct; the first expression is equivalent to 10*b*, and that is twice the second expression, which is equivalent to 5*b*. Doubling would double the difference as well.

Use the TNS activity to check whether the following are equivalent. If they are not, change one of the expressions to make them equivalent.

- $-(-a)^2$ ***and*** $(a)^2$

Answer: They are not equivalent; eliminating the negative sign in the first expression or putting it inside the parentheses in the second expression will make the expressions equivalent.

- $-(-a)^3$ ***and*** $a(-a)^2$

Answer: The expressions are equivalent.

- $-ab$ ***and*** $-(-a)(-b)$

Answer: The expressions are equivalent.

Use the TNS activity to check which of the following is equivalent to $7-2(3-8a)$. Think about the order of operations and give an explanation for the conclusion you made using the evidence from the activity.

a. $5(3-8a)$

b. $7-2(-5a)$

c. $7-6-16a$

d. $7-6+16a$

Answer: *a* is not equivalent because you have to multiply the value of the expression in the parentheses by 2 before you subtract from 7. *b* is not correct because $3-8a$ is not equivalent to -5*a*. *c* is not equivalent because the subtraction sign before the 2 refers to the whole expression in the parentheses. *d* is correct.

Properties of operations can be used to create equivalent expressions. In each case, apply the given property or procedures, then check using the TNS activity to see that you were correct.

- ***Distributive property of multiplication over addition: $12a-3(a+5)$***

Answers may vary. Possible answers: $12a-3a-15$; $9a-15$; $3(4a-(a+5))$

Class Discussion (continued)

- ***Regrouping and then combining:*** $\left(\frac{1}{3}a+b\right)-\left(\left(\frac{5}{6}\right)a-3b\right)$

  Answers may vary. Possible answers: $\frac{1}{2}a+4b$

- ***Distributive property of multiplication over addition and then combining:*** $3(b+4)-8(b-2)$

  Answers will vary. Possible answer: $-5b+28$

- ***Distributive property of multiplication over addition:*** $\left(\frac{1}{2}\right)(2a-1)-\left(\frac{3}{2}\right)(2a-1)$

  Answers will vary. Possible answers: $-(2a-1)$; $\left(a-\frac{1}{2}\right)-\left(3a-\frac{3}{2}\right)$

Explain how the structure of each of the following expressions is different from the expression $\left(-\frac{3}{4}\right)(2a-4)$ ***and decide whether any of the expressions are equivalent. Check your thinking using the TNS activity.***

- $\frac{((-3)(2a-4))}{4}$

  Answer: The $-\frac{3}{4}$ times the parentheses has been rewritten as the product of -3 and the parentheses divided by 4. Equivalent.

- $\left(-\frac{3}{2}\right)(a-2)$

  Answer: The 2 has been factored out of the 2*a*-4 and grouping the two multipliers, $\left(-\frac{3}{4}\right)(2)=-\frac{3}{2}$. Equivalent

- $\frac{(-6a+12)}{4}$

  Answer: The -3 has been distributed into the sum in the parentheses. Equivalent

- $\left(-\frac{3}{2}\right)a+3$

  Answer: The $-\frac{3}{4}$ has been distributed into the sum in the parentheses and the computation of $\left(-\frac{3}{4}\right)(-4)$ is 3. Equivalent

Class Discussion (continued)

Which of the following represents an increase of 10% for an item that cost b dollars? Explain your reasoning and use the TNS activity to check.

a. $\left(\frac{1}{10}\right)b$

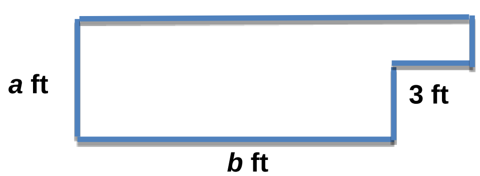
b. $1\frac{1}{10}b$

c. $b+\left(\frac{1}{10}\right)b$

d. $\frac{1}{10}+b$

Answer: *b* and *c* both represent an increase because you can think of *b* as 1*b* and then distribute (factor) out the *b* to get $\left(1+\frac{1}{10}\right)=1\frac{1}{10}b$

Shep decided to fence in his back yard for his dog. Which of the expressions could be used to find the length of fencing he will need if his back yard is shaped like the following:



a. $2(a+b+3)$

b. $2a+2b+3$

c. $2a+2(b+3)$

d. $2(a+b)+6$

Answer: *a*, *c*, and *d* are all equivalent and will give the perimeter of the backyard.

Student Activity Questions—Activity 1

1. Make a conjecture about which, if any, of the following are equivalent expressions and why. Think about the order of operations, in particular subtracting a quantity, and what that means. Use the TNS activity to check your conjecture.

a. $\left(\frac{5}{3}\right)a+\left(4-\frac{8}{3}a\right)$

b. $\frac{5}{3}a-\left(4-\frac{8}{3}a\right)$

c. $\frac{5}{3}a-\left(\frac{8}{3}a-4\right)$

d. $\frac{5}{3}a-2\left(2-\frac{4}{3}a\right)$

Answer: *a* and *c* are equivalent; *b* and *d* are equivalent. Reasons will vary but might include statements such as: “subtracting an expression like $\frac{8}{3}a-4$ is the same as adding the opposite of the expression, $-\frac{8}{3}a+4$”; “distributing out a 2 from $4-\frac{8}{3}a$ produces $2\left(2-\frac{4}{3}a\right)$ the product of 2 and $\left(2-\frac{4}{3}a\right)$ is $4-\frac{8}{3}a$.

2. Identify the following statements as true or false. Use the TNS activity to support your reasoning.

a. **In the expression $2a+3b$, the variables *a* and *b* must always have different values.**

Answer: False, *a* and *b* can have the same or different values: $a=5$ and $b=5$ means $2a+3b$ would have value 25, but when $a=5$ and $b=4$, the value of $2a+3b$ would be 22.

b. **$2a+3b$ is equivalent to 5*ab.***

Answer: In general, this is false but if $a=b=1$, it will be true.

c. **$\frac{3}{4}b-\left(-\frac{1}{4}\right)b$ is equivalent to *b.***

Answer: This is true; the expression is equivalent to 1*b*.

d. **If *a* and *b* have the same value, then the expressions 2*a* and 3*b* will never have the same value.**

Answer: This is false. If the values of *a* and *b* are both 0, then 2*a* and 3*b* will each have value 0*.*

Student Activity Questions—Activity 1 (continued)

3. For each of the following, find an equivalent expression of the given form where *c*, *d,* and *e* are rational numbers. Use the TNS activity to check your thinking.

a. of the form $cx+d$ **:** $\left(\frac{1}{2}\right)(x-7)-\frac{1}{4}x$

Answer: Sample answer: $\frac{1}{4}x-\frac{7}{2}$

b. of the form $c(x+d)$**:** $3(x-7)-11(x-7)$

Answer: Sample answer: $-8(x-11)$

c. of the form $c+dx$ **:** $\frac{1}{2}x-\frac{3}{4}-\frac{2}{3}x+\frac{1}{8}$

Answer: Sample answer: $-\frac{5}{8}-\frac{1}{6}x$

d. of the form $c(dx+e)$**:** $5x-15(x-3)$

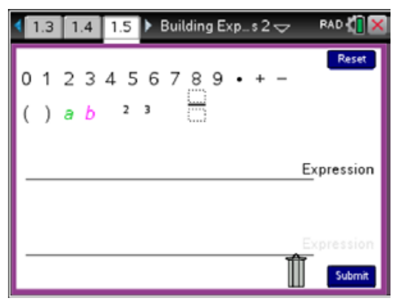
Answers may vary. Using only integers, you might get $5(-2x+9)$.

Part 2, Page 1.5

Focus: Identify patterns in a table and connect these patterns to the relationship between two expressions.

Page 1.5 functions in the same way as page 1.3, but **Submit** displays a table.

Use the keypad and the up/down arrows to select values for *a* or *b*, then **Enter** to display the values in the table.



Class Discussion

Enter the expression 3a and Submit. Delete the value for b. In each of the following, use the columns in the table to describe what happens to the value of the expression 3a under the given change. Explain why your answer makes sense.

- ***Increase a by 4 and Enter. Repeat the process several times using different values of a. If a increases by 4, then 3a changes by…***

  Answer: The value of the expression increases by 12 because $3(a+4)$ is the same as $3a+12$ by the distributive property of multiplication over addition.

Class Discussion (continued)

- ***Select* Edit *and without changing the expression,* Submit*. This should clear the values in the table. If a decreases by 5, then 3a changes by…***

Answer: The value of the expression decreases by 15 because $3(a-5)=3a-15$ by the distributive property of multiplication over addition.

- ***If your choices for a are negative values, $-3a$ will be greater than, less than or equal to 0?***

Answer: The value of $-3a$ will be greater than 0 because the product of -3 and a negative number will be positive number.

Enter each of the following expressions, then* Submit*. Enter different values for a and or b then inspect the table to determine if one expression is related to the other in any way. (For an expression with only one variable, delete the value for the other and* Enter*.) In each case, explain why your answer makes sense.

Note: students might need to use values that are from -4 to 4 in order to see the patterns in the table.

- ***20a and 4a***

Answer: 20*a* is five times the expression 4*a* because $5(4a)=20a$.

- $\left(\frac{1}{3}\right)ab$ ***and 3ab***

Answer: 3*ab* is 9 times $\left(\frac{1}{3}\right)ab$ because 9 $\left(\frac{1}{3}\right)ab=\left(\frac{9}{3}\right)(ab)=3ab$.

- $-a+b$ ***and*** $a-b$

Answer: One is -1 times the other because $-1(a-b)=-1(a)-(-1b)=-a+b$. The expressions are opposites.

- $6a-3b$ ***and*** $2a-b$

Answer: $6a-3b$ is 3 times $2a-b$ because $3(2a-b)=6a-3b$ by the distributive property of multiplication over addition.

Student Activity Questions—Activity 2

1. **On page 1.5 enter the two given expressions and Submit. Generate values for the table and use them to answer the question: As *a* goes from 1 to 500 and beyond, which of the two expressions has the larger value? Explain why your answer makes sense in each case.**

   a. **$200+20a$ and $200+50a$**

   Answer: $200+50a$ because at $a=6$, the two expressions are equal. For $a>6$, $20+50a$ increases by 50 for each unit change in *a*, while $200+50a$ only increases by 20 and so the value of $20+50a$ will be larger.

   b. **$2a$ and a^2**

   Answer: a^2 because at $a=2$, the expressions are equal but then a^2 increases faster.

   c. **$-5a-10$ and $-10a+10$**

   Answer: They are even at 4, then as *a* increases the value of $-5a-10$ becomes larger than the value of $-10a+10$.

   d. **$\frac{a^2}{a^3}$ *and* $\frac{1}{(5a)}$**

   Answer: $\frac{1}{(5a)}$ will be smaller than $\frac{a^2}{a^3}=\frac{1}{a}$ because as the denominator of a unit fraction increases, the size of the fractional part of the unit decreases.

2. **The cost to belong Bey's music club is $14, and you can download a song for $2. The cost to belong to Mado's music club is $8, and you can download a song for $3.**

   a. **If you downloaded 4 songs, which music club would be cheaper? Explain your thinking.**

   Answer: Bey's: $14+4(2)=22$; Mado's $8+4(3)=20$

   b. **Let *a* represent the number of songs you downloaded. Write expressions for downloading "*a*" songs from each club. Enter the expressions on page 1.5 and Submit.**

   Answer: Bey: $14+2a$ and Mado: $8+3$

   c. **Use the table to determine which music club will be the least expensive.**

   Answer: For 1 to 5 downloads, Mado will be the least expensive; for 6 downloads both music clubs will cost the same, and for more than 6 downloads, Bey's will be the least expensive.

Part 3, Pages 1.3 and 1.5

Focus: Move fluently among equivalent expressions involving squares and cubes.

Class Discussion

Refer to page 1.3 when working through the following questions. Enter the two expressions and submit. Are the two equivalent? Use the TNS activity and then explain the mathematics behind your conclusion.

- $\left(-b^2\right)$ ***and*** $-\left(b^2\right)$

  Answer: The expressions are not equivalent because when you square $-b$ you get a positive b^2. The expressions are opposites.

- $(-3b)^2(-3b)^3$ ***and*** $(3b)^2(3b)^3$

  Answer: The expressions are not equivalent because for $b>0$, the first is always negative and the second is always positive, while for $b<0$, the first is always positive and the second always negative. The two expressions are opposites.

- $-(ab)^2$ ***and*** $(-a)^2(-b)^2$

  Answers: The absolute values are the same, but the first is negative and the second is positive so the expressions are not equivalent.

Are any of the following expressions equivalent? Use the TNS activity to check your answer. For each, explain mathematically why your conclusion is valid.

a. $\left(\frac{2}{3}\right)b(3b)$

b. $\frac{2}{3}b(b+3)$

c. $\left(\frac{2}{3}b\right)b+3$

d. $\left(\frac{2}{3}\right)b+(2b)\left(\frac{1}{3}\right)$

Answer: *b* and *d* are equivalent because of the distributive property of multiplication over addition where the (2*b*) has been distributed to the terms b and 3 in the parentheses. *a* is not equivalent to the others because adding 3 is not the same as multiplying by 3, so $b+3$ is not 3*b*. *c* is not equivalent to any of the others because it does not involve the 3 with a factor of *b* as the others do.

Class Discussion (continued)

Finding the product of two factors that are algebraic expressions is an important skill. Find an expression without any parentheses that is equivalent to each of the following.

- $x(3x-4)$ — Answers may vary. One response might be $3x^2-4x$.
- $2x(3x-4)$ — Answers may vary. One response might be $6x^2-8x$.
- $-2x(4-3x)$ — Answers may vary. One response might be $6x^2-8x$.
- $x(3x-4)-2x$ — Answers may vary. One response might be $3x^2-6x$.

Which of the following are equivalent expressions?

Answer: *c* and *d* each have two equivalent expressions.

a. $(a-b^2)$ and a^2-b^2

b. $(a+b)^2$ and a^2+b^2

c. $(a+b)^2$ and a^2+b^2+2ab

d. $(a+b)(a-b)$ and a^2-b^2

Use the TNS activity to determine what values for k will make the two expressions equivalent.

- a^2+ak-6 and $(a-3)(a+2)$ — Answer: $k=-1$
- $3a^2+ak-6$ and $(3a-2)(a+3)$ — Answer: $k=5$
- $3a^2+ak-6$ and $(3a-3)(a+2)$ — Answer: $k=3$
- $3a^2+ak+6$ and $(3a-2)(a-3)$ — Answer: $k=-11$

Class Discussion (continued)

Refer to page 1.5 when working through the following questions.

***Enter $2a^3 - 8a$ for the first expression and $a^2 - 4$ and select* Submit.**

- ***Let $a = 3$ Is there a relationship between Expr1 and Expr2 and the value of a?***

  Answers may vary. $3(5) = 15$ which goes into 30 twice so 3(Expr2) goes into Expr1 twice. Or Expr1 divided by Expr2 is twice the value of *a*. $\frac{30}{5} = 6$, which is twice 3.

- ***Is the relationship you discovered valid for $a = 4$, 5, and 6?***

  Answers may vary. For either of the two responses in the question above the answer would be yes.

- ***Mica says that for any a, $\frac{Expr1}{a(Expr2)} = 2$. Levi says that $Expr1 = 2a(Expr2)$. Check the values in the table to see if you agree with Mica or Levi. Explain your reasoning.***

  Answers may vary. They are both right. For example, when $a = 4$, $\frac{96}{4}(12) = 2$ and $96 = 2(4)(12)$

- ***Write Mica's and Levi's claims using the rules for Expr1 and Expr2: $2a^3 - 8a$ and $a^2 - 4$. Explain how the claims are related.***

  Answers: Mica's claim: $\frac{2a^3 - 8a}{a(a^2 - 4)} = 2$; Levi's claim: $2a^3 - 8a = 2a(a^2 - 4)$. The two claims are the same because they describe the relationship between multiplication and division: $\frac{x}{y} = z$ is the same as $x = yz$.

***Enter $\frac{2}{3}a - \frac{1}{2}b$ for the first expression and $4a - 3b$ for the second.* Submit.**

- ***What do you notice about the values for Expr 1?***

  Answer: They all have a denominator of 6 except for those that might have been rewritten with a denominator of 2 or 3. Expr 2 is the numerator of all of the fractions.

- ***Predict what the value of each expression will be if $a = 30$ and $b = 20$. Use the TNS activity to check your answer.***

  Answer: 10 and 60.

- ***Write an equation showing how Expr 1 and Expr 2 are related.***

  Answer: $\frac{2}{3}a - \frac{1}{2}b = \frac{(4a - 3b)}{6}$

Class Discussion (continued)

- ***Rewrite your answer to the question above so the equation does not have any division.***

  Answer: $6\left(\frac{2}{3}a-\frac{1}{2}b\right)=(4a-3b)$

Enter a^2-b^2 as the first expression and $a-b$ as the other. Submit. Let $a=2, b=1$, Enter. Then increase both a and b in steps of 1.

- ***What do you notice about the values in Expr1? Why does this make sense?***

  Answer: They are consecutive odd numbers because each time you add one to *a* and *b*, the difference in the squares is increased by a factor of $2(a-b)$ which in this case will always be 2.

  Since the original difference was odd, each of the consecutive differences will also be odd.

- ***Inspect the columns to see how Expr1 is related to the values you entered.***

  Answer: Expr1 is always $a+b$

- **Edit,** ***then*** **Submit** ***without changing anything. This time let $a=3, b=1$,*** **Enter*****. Then increase both a and b in steps of 1. How did the values for Expr1 change?***

  Answer: This time Expr1 is $2(a+b)$

- ***Repeat the process from the question above beginning with $a=4$ and $b=1$. Predict what the values for Expr1, then check using the TNS activity.***

  Answer: $3(a+b)$

Enter Expr1 as a^2-b^2 and Expr2 as $a+b$ and **Submit**.

- ***Enter a as 5 and b as 3. Describe a relationship you see among the numbers in the table.***

  Answers may vary. Expr2 goes into Expr1 twice; $\frac{16}{8}=2$. Some may notice that the difference between *a* and *b* is 2.

- ***Add one to a and to b and Enter. Repeat this action several times. Does the relationship you described in the question above work for the new values?***

  Answers may vary. For the two noted in the response above, the answer is yes.

- ***Earl claims that Expr1 divided by Expr2 is the difference between a and b. Do you agree or disagree with Tami? Explain your reasoning.***

  Answers may vary. Arguing from the numbers in the table support Earl's claim, yes , Expr1 divided by Expr2 is always the difference between a and b.

Class Discussion (continued)

• ***Tami claims that $a^2 - b^2 = (a-b)(a+b)$. Use Expr1 and Expr2 to help decide whether you agree or disagree with Tami. Explain your reasoning.***	Answers may vary: Tami is correct because the division you can see in the table can be rewritten as multiplication.

Deeper Dive — Page 1.5

On page 1.5, enter $3a+2b$ for the first expression and $2a+b$ for the second expression. **Submit.**

• ***Enter. What values do you see in the first row of the table?***	Answer: 2, 2, 12, 6.
• ***Vary the values of a and b with at least one of the numbers an odd number, then Enter. Describe the values you see in the table.***	Answer will vary. One example might be 5, 3, 21, 13.
• ***Continue to vary a and b, keeping one of the values odd. What relationship do you see between the numbers in a given row in the table?***	Answer: The difference between the entries in column 3 and column 4 is the sum of the entries in column 1 and column 2.
• ***Write an expression describing the relationship between the two numbers.***	Answer: $(3a+2b)-(2a+b)=a+b$.

Enter $4a+3b$ for the first expression and $3a+2b$ for the second expression.

• ***When will the expressions give you the same value?***	Answer: When *a* and *b* are opposites.
• ***Vary the input for a and b, keeping one of the values as an odd number. Write an equation for the relationship between the two expressions***.	Answer: $(4a+3b)-(3a+2b)=a+b$.
• ***Make up another two expressions that have the same relationship as the one you wrote in the question above.***	Answer: Any two expressions where the coefficients of *a* and *b* differ by 1 and the coefficients of *b* in the first expression is 1 less than the coefficient of *a* will have a difference of $a+b$**.**
• ***Jena claims that b is a factor of the expression $3b+b^2$. Use the TNS activity to decide whether Jena is correct. Explain how the TNS activity helped you decide.***	Answers will vary. By the distributive property of multiplication over addition, $b(3+b)=3b+b^2$.

Deeper Dive — Page 1.5 (continued)

Let expression one be 2ab and expression two be $a^2 - b^2$. Enter values for a and b that are 1 unit apart with a going from 2, to 3, … to 8 and b one less than a.

- ***Describe the values for Expr2. What do these numbers have in common?***

  Answers may vary: The numbers are all even or all multiples of 4; 4, 12, 24, 40, …

- ***Divide each value in Expr2 by 4. What sequence do you get?***

  Answer: You get 1, 3,6,10, …. , which is the sequence of triangular numbers.

- ***Describe the values for Expr1. How are the values related to a and b?***

  Answer: The numbers are consecutive odd numbers, and each value is the sum of the *a* and *b* that generated the expression.

Songe wondered if the values in the table were connected to Pythagorean Triples.

- ***Remember the Pythagorean Theorem and use it to find positive whole numbers that can be the length of the sides of a right triangle.***

  Answers may vary. 3, 4, 5 or 5, 12, 13 are two examples.

- ***Do you think Songe is right? Test out some other numbers to check your thinking.***

  Answers: Yes, the numbers for Expr1 and Expr2 are the legs of right triangles.

- ***Saul claimed that every single odd number would be in a Pythagorean triple. Do you think Saul is right? Why or why not?***

  Answers may vary. Yes, because you can keep going getting more and more *a*s and *b*s one unit apart that will keep giving Pythagorean Triples where the odd number is the length of the smallest leg of the right triangle.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Consider each of the following expressions. In each case, does the expression equal 2*x* for all values of *x*? Indicate YES or NO for each expression.

   a. 2 times x

   b. *x* plus *x*

   c. *x* times *x*

National Assessment of Educational Progress Question ID: 2007-8M9 #2 M0757CL

Answer: yes for a. 2 times x and b. x plus x . No for c. x times x

2. In the equation $y = 4x$, if the value of *x* is increased by 2, what is the effect on the value of *y* ?

   a. It is 8 more than the original amount.

   b. It is 6 more than the original amount.

   c. It is 2 more than the original amount.

   d. It is 16 times the original amount.

   e. It is 8 times the original amount.

National Assessment of Educational Progress Question ID: 2005-8M3 #10 M067401

Answer: a. It is 8 more than the original amount.

3. Indicate whether each expression in the table is equivalent to $\frac{1}{2}x-1$, $x-\frac{1}{2}$, or not equivalent to $\frac{1}{2}x-1$ or $x-\frac{1}{2}$.

	Equivalent to $\frac{1}{2}x-1$	Equivalent to $x-\frac{1}{2}$	Not equivalent to $\frac{1}{2}x-1$ or $x-\frac{1}{2}$.\
$\frac{2}{3}\left(\frac{3}{4}x-\frac{3}{2}\right)$			
$(2x+1)-\left(x+\frac{3}{2}\right)$			

PARCC Math Spring Operational 2015 Grade 7 released item VF888892

Answer:

	Equivalent to $\frac{1}{2}x-1$	***Equivalent to*** $x-\frac{1}{2}$	***Not equivalent to*** $\frac{1}{2}x-1$ ***or*** $x-\frac{1}{2}$.
$\frac{2}{3}\left(\frac{3}{4}x-\frac{3}{2}\right)$	***yes***	***no***	
$(2x+1)-\left(x+\frac{3}{2}\right)$	***no***	***yes***	

4. Jordan's dog weighs p pounds. Emmett's dog weighs 25% more than Jordan's dog. Which expressions represent the weight, in pounds, of Emmett's dog. Select each correct answer.

   a. $2\frac{1}{4}p$ b. $1\frac{1}{4}p$ c. $p+\frac{1}{4}$ d. $p+1\frac{1}{4}$ e. $p+\frac{1}{4}p$

   adapted from PARCC Math Spring Operational 2015 Grade 7 released item VF541775

   Answer: b. $1\frac{1}{4}p$ ***and e.*** $p+\frac{1}{4}p$

5. Determine which expression is equivalent to $\frac{3}{4}-x\left(\frac{1}{2}-\frac{5}{8}\right)+\left(-\frac{3}{8}x\right)$

   a. $\frac{3}{4}x$ b. $\frac{1}{2}x$ c. $\frac{1}{8}-\frac{7}{8}x$ d. $\frac{3}{4}-\frac{1}{4}x$

   PARCC Math Spring Operational 2015 Grade 7 released item MC21535

   Answer: d. $\frac{3}{4}-\frac{1}{4}x$

6. Find the value of p so the expression $\frac{5}{6}-\frac{1}{3}n$ is equivalent to $p(5-2n)$.

   adapted from Smarter Balance Practice Test, Grade 7 Mathematics Item 1837

   Answer: $\frac{1}{6}$

7. Which expression is equivalent to $-8(10x-3)$?

   a. $-80x+24$ b. $-80x-24$ c. $-80x+3$ d. $-80x-3$

   Smarter Balance Practice Test, Grade 7 Mathematics Item 1836

   Answer: a. $-80x+24$

Building Concepts: Building Expressions in Two Variables

TEACHER NOTES

Student Activity Solutions

In these activities you will identify equivalent expressions involving rational numbers. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Make a conjecture about which, if any, of the following are equivalent expressions and why. Think about the order of operations, in particular subtracting a quantity, and what that means. Use the TNS activity to check your conjecture.

   a. $\left(\frac{5}{3}\right)a+\left(4-\frac{8}{3}a\right)$

   b. $\frac{5}{3}a-\left(4-\frac{8}{3}a\right)$

   c. $\frac{5}{3}a-\left(\frac{8}{3}a-4\right)$

   d. $\frac{5}{3}a-2\left(2-\frac{4}{3}a\right)$

   Answer: a and c are equivalent; b and d are equivalent. Reasons will vary but might include statements such as: "subtracting an expression like 8/3 a-4 is the same as adding the opposite of the expression, $-\frac{8}{3}a+4$ *"; "distributing out a 2 from 4-8/3 a produces* $2\left(2-\frac{4}{3}a\right)$ *the product of 2 and* $\left(2-\frac{4}{3}a\right)$ *is* $4-\frac{8}{3}a$

2. Identify the following statements as true or false. Use the TNS activity to support your reasoning.

   a. In the expression $2a+3b$, the variables *a* and *b* must always have different values.

   Answer: False, a and b can have the same or different values: $a=5$ *and* $b=5$ *means* $2a+3b$ *would have value 25, but when* $a=5$ *and* $b=4$*, the value of* $2a+3b$ *would be 22.*

   b. $2a+3b$ is equivalent to 5*ab.*

   Answer: In general, this is false but if $a=b=1$*, it will be true.*

   c. $\frac{3}{4}b-\left(-\frac{1}{4}\right)b$ is equivalent to *b.*

   Answer: This is true; the expression is equivalent to 1b.

   d. If *a* and *b* have the same value, then the expressions 2*a* and 3*b* will never have the same value.

   Answer: This is false. If the values of a and b are both 0, then 2a and 3b will each have value 0.

3. For each of the following, find an equivalent expression of the given form where *c*, *d,* and *e* are rational numbers. Use the TNS activity to check your thinking.

   a. of the form $cx+d$: $\left(\frac{1}{2}\right)(x-7)-\frac{1}{4}x$

   Answer: Sample answer: $\frac{1}{4}x-\frac{7}{2}$

   b. of the form $c(x+d)$: $\mathbf{3}(x-\mathbf{7})-\mathbf{11}(x-\mathbf{7})$

   Answer: Sample answer: $-8(x-11)$

   c. of the form $c+dx$: $\frac{1}{2}x-\frac{3}{4}-\frac{2}{3}x+\frac{1}{8}$

   Answer: Sample answer: $-\frac{5}{8}-\frac{1}{6}x$

   d. of the form $c(dx+e)$: $5x-15(x-3)$

   Answers may vary. Using only integers, you might get $5(-2x+9)$.

Activity 2 [Page 1.5]

1. On page 1.5 enter the two given expressions and Submit. Generate values for the table and use them to answer the question: As *a* goes from 1 to 500 and beyond, which of the two expressions has the larger value? Explain why your answer makes sense in each case.

   a. $200+20a$ and $200+50a$

   Answer: $200+50a$ *because at* $a=6$ *, the two expressions are equal. For* $a>6$ *,* $20+50a$ *increases by 50 for each unit change in a, while* $200+50a$ *only increases by 20 and so the value of* $20+50a$ *will be larger.*

   b. $2a$ and a^2

   Answer: a^2 *because at* $a=2$ *, the expressions are equal but then* a^2 *increases faster.*

   c. $-5a-10$ and $-10a+10$

   Answer: They are even at 4, then as a increases the value of $-5a-10$ *becomes larger than the value of* $-10a+10$ *.*

   d. $\frac{a^2}{a^3}$ *and* $\frac{1}{(5a)}$

   Answer: $\frac{1}{(5a)}$ *will be smaller than* $\frac{a^2}{a^3}=\frac{1}{a}$ *because as the denominator of a unit fraction increases, the size of the fractional part of the unit decreases.*

2. The cost to belong Bey's music club is $14, and you can download a song for $2. The cost to belong to Mado's music club is $8, and you can download a song for $3.

   a. If you downloaded 4 songs, which music club would be cheaper? Explain your thinking.

   Answer: Bey's: $14+4(2)=22$ *; Mado's* $8+4(3)=20$

   b. Let *a* represent the number of songs you downloaded. Write expressions for downloading "*a*" songs from each club. Enter the expressions on page 1.5 and Submit.

   Answer: Bey: $14+2a$ *and Mado:* $8+3$

   c. Use the table to determine which music club will be the least expensive.

   Answer: For 1 to 5 downloads, Mado will be the least expensive; for 6 downloads both music clubs will cost the same, and for more than 6 downloads, Bey's will be the least expensive.