Building Concepts: Equations and Operations TEACHER NOTES

Lesson Overview

Algebraic Focus: Algebraic actions on equations result in new equations. Which of these actions will preserve the solution from the original equation as the solution to the new equation?

In this lesson students investigate the relationship between the solution to an equation and the solution to a new equation formed by adding, subtracting, multiplying, or dividing one or both sides of the original equation by a numerical value. Students develop an understanding of “solution-preserving moves” by thinking carefully about the structure of relationships expressed by an equation and how to exploit that structure to find a solution.

You can add or subtract a number from both sides of a given equation and the new equation will shave the same solution as the original equation

Learning Goals

1. Associate an addition problem with a related subtraction problem and a multiplication problem with a related division problem;
2. recognize that adding the same number to both sides of an equation or multiplying both sides of an equation by the same nonzero number will result in a new equation having the same solution set;
3. identify solution-preserving mathematical moves related to equations and moves that do not preserve solutions;
4. describe a strategy for finding a solution for an equation of the form $x+p=q$.

Prerequisite Knowledge

Equations and Operations is the fifth lesson in a series of lessons that explore the concept of expressions. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *Building Expressions* and *What is an Equation?* Students should understand:

- how to distinguish between expressions and equations;
- what it means to have a solution to an equation;
- the associative and commutative properties of addition and multiplication.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **equation:** a statement in which two expressions are equal
- **variable:** a letter that represents a number in an expression
- **solution:** a number that makes the equation true when substituted for the variable

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Equations and Operations_Student.pdf
- Equations and Operations_Student.doc
- Equations and Operations.tns
- Equations and Operations_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

Solving an equation is a process of reasoning to find the numbers, if any, that make the equation true, which can include checking if a given number is a solution. In grade 6 students begin to think about solutions to equations and how to find them in some strategic way. If the original equation has a solution, solving the equation is a search for that solution using a variety of reasoning strategies. These strategies include considering solution-preserving algebraic actions that can be used on equations. For example, adding the same quantity to both sides of an equation produces a new equation having the same solution. The focus in this lesson is not on developing a formal procedure for finding solutions but rather on thinking carefully about the structure of the relationships expressed by an equation and how to exploit that structure to find a solution.

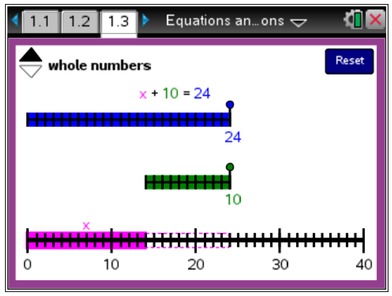
In Activity 4, *What is an Equation?*, students developed the concept of equation and of a solution to that equation. Activity 5 investigates the relationship between the solution to an original equation and the solution to a new equation formed by adding, subtracting, multiplying or dividing one or both sides of the original equation by a numerical value. Some of the problems involve shifting from an arithmetic problem to a corresponding algebraic problem to show the connection between procedures for solving equations and the reasoning behind those procedures. Because the activity involves fundamental concepts related to solving equations, working through the questions might take two or three class periods.

The lesson engages students in working with equations of the form $x+a=b$, $x-a=b$, and $ax=b$ where a, b and x are positive numbers. It is important for them to understand that $a+x=b$, $b=x+a$ and $b=a+x$ are equivalent forms for the equation $x+a=b$.

Part 1, Page 1.3

Focus: Students think about “solution-preserving moves”, which is a general way to talk about adding, subtracting, multiplying or dividing both sides of an equation by some value.

On page 1.3, students change the value of the equation and lengths of the segments by grabbing and dragging the handle on the blue or green segment.



TI-Nspire Technology Tips

menu accesses page options.

tab toggles between the green and blue segments and, when fractions are chosen, the denominator setting.

ctrl del resets the page.

Scale changes step of the scale from whole numbers to fractions or decimal. Using the up and down arrow also adjusts the scale on the number line. When fractions are chosen, the left and right arrows adjust the denominator value.

Adjust Segment allows the length of the blue segment (total) or the green segment (constant) to be changed using the arrow keys.

Reset resets the page.

To create a specific equation, $x + a = b$, first drag the green handle to set a, then adjust the blue segment to equal b.

Class Discussion

Teacher Tip: The first questions are designed to help students understand how the file behaves. The questions push students to think about “solution-preserving moves”, which is a general way to talk about adding, subtracting, multiplying or dividing both sides of an equation by some value. Students should figure out which moves preserve solutions from the original equation to the new equation and which do not. The remaining questions introduce students to the solution-preserving move of adding the same value to both sides of an equation.

Have students…	Look for/Listen for…
Increase the length of the green segment by moving the dot one unit to the right.	
• ***Describe what happens to the pink and blue segments.***	Answer: The blue segment increases by one unit; the pink segment does not change.
• ***Describe what happens to the equation.***	Answer: The equation goes from $x + 10 = 24$ to $x + 11 = 25$. Increasing the green segment by 1 makes the left side of the equation 1 larger and the right side of the equation, the 24, increase by one unit as well.

Class Discussion (continued)

• ***Reset the page. Predict what you think the equation will be if you make the green segment 5 units longer and what the value of x will be. Use the file to check your prediction.***	Answer: The new equation will be $x+15=29$; the value of *x* will still be 14.
• ***Reset the page. Predict what you think the equation will be if you make the green segment 5 units shorter and what the value of x will be. Use the file to check your prediction.***	Answer: The new equation will be $x+5=19$; the value of *x* will still be 14.
Reset the page. Increase the length of the blue segment by moving the dot 6 units to the right.	
• ***Describe what happens to the pink and green segments.***	Answer: The green segment does not change; the pink segment increases in length (by 6).
• ***Describe what happens to each side of the equation and to the value of x.***	Answer: The equation changes from $x+10=24$ to $x+10=30$, and the value of *x* increases by 6 units from $x=14$ to $x=20$.
• ***Reset the page. Predict what you think the equation will be if you make the blue segment 10 units longer and what the value of x will be. Use the file to check your prediction.***	Answer: The new equation will be $x+10=34$; the value of *x* will be 24.
• ***Reset the page. Predict what you think the equation will be if you make the green segment 10 units shorter and what the value of x will be. Use the file to check your prediction.***	Answer: The new equation will be $x+0=14$; the value of *x* will be 14.
Reset the page. Create the equation $x+21=37$ by first dragging the green segment to have length 21, and then the blue segment to have length 37.	
• ***Remember what the solution to an equation represents. How is the solution to the equation reflected in the segments? How do you know?***	Answer: The solution for the equation $x+21=37$ is the length of the pink segment; in this case $x=16$, because that makes a true statement, $16+21=37$.
• ***Reset. Drag the green segment until the constant term on the left side of the equation is 13. Describe the new equation.***	Answer: The new equation will be $x+13=29$.

Class Discussion (continued)

- ***What is the solution for the new equation? Explain why your answer is correct.***

  Answer: The solution for $x + 13 = 29$ is 16 because $x = 16$ makes a true statement: $16 + 13 = 29$.

Create the equation $x - 7 = 29$.

- ***How long is the pink segment?***

  Answer. 36 units

- ***What is the solution to the equation?***

  Answer. $x = 36$

- ***Tanya says solving an equation that has a number subtracted from x is like solving an equation with a number added to x except now you can add that number to both sides of the equation. Do you agree or disagree with Tanya? Give an example to support your reasoning.***

  Answers may vary. Tanya is right because a subtraction problem can be written as an addition problem ($15 - 5 = 10$ can be written as $15 = 10 + 5$); Tanya is right because when you add and subtract the same number you get 0 ($x - 5 + 5 = 10 + 5$, so $x = 15$)

> **Teacher Tip:** Select several students to share their answers to the following question during a whole class discussion to ensure that everyone understands how to create new equations that are "equivalent" to the original equations because they have the same solution.

Describe any "solution-preserving moves" you can use to create a new equation from one you are given. Find examples from the questions in Part 1 that support your reasoning.

Answers will vary. Students might note that you can add or subtract a number from both sides of a given equation and the new equation will have the same solution as the original equation. They could use almost any of the prior questions; the two previous questions might be cited.

Student Activity Questions—Activity 1

1. Adjust the length of the blue and green segments to create the equation $x + 17 = 32$. Answer each of the following. Explain your reasoning in each case.

a. If you make the green segment larger, what happens to the equation?

Answer: The amount you increase the green segment is added to the right side of the equation as well as to the constant term on the left side.

b. What happens to the pink segments and to the solution to the equation?

Answer: The pink segments remain the same size, 15, and the solution, the value for *x*, is always the same, $x = 15$.

c. If you make the green segment smaller, what happens to the equation? To the solution to the equation?

Answer: The amount is subtracted from the constant term on the left and from the 32. The solution stays the same, $x = 15$.

Student Activity Questions—Activity 1 (continued)

2. **Reset the page. Create the equation $x+17=32$ again.**

   a. **If you make the blue segment larger, what happens to the equation?**

   Answer: The amount you increase the blue segment is added to the right side of the equation, and *x* increases in length by that same amount.

   b. **What happens to the pink segments and to the solution to the equation?**

   Answer: The pink segments increase in length. The solution, the value for *x*, gets larger as you increase the right side of the equation by the amount of that increase.

   c. **If you make the blue segment smaller, what happens to the equation? To the solution to the equation?**

   Answer: The amount is subtracted from the 32, and the rest of the equation stays the same; for example, when you make the blue segment 3 units shorter, the equation becomes $x+17=29$. The value for *x*, which is the solution, gets smaller by the same amount that the right side increased, $x=12$ for the example.

3. **Students are discussing equations of the form $x+a=b$ where *a* and *b* are given numbers. Decide whether you agree or disagree with the statements some of them made. Give a reason for your answer.**

   a. **Sandra said that adding 2 to the left side of the equation and 3 to the right side of the equation will make a new equation whose solution is 1 more than the solution to the original equation.**

   Answer: Sandra is right because you had one more on the right side so *x* would have to be one greater.

   b. **Mark said that the solution to the equation will be $x=b-a$.**

   Answers may vary. Mark is right because $x=b-a$ makes a true statement: $(b-a)+a=b$. Or Mark is correct because an addition problem such as $5+7=12$ can be written as a subtraction problem in two different ways, $5=12-7$ or $7=12-5$.

   c. **Patou said that if you subtract 11 from both sides of an equation, the value of *x* for the new equation will be 11 less than the value of *x* in the original equation.**

   Answer: Patou is not correct because if you subtract 11 from both sides, the amount you need to make the sides the same will not change.

   d. **Mabel said that adding the same number to both sides of an equation will not change the solution.**

   Answers may vary. Mabel is correct because if you have $x+5=12$ and add a number *c* to both sides, the new equation will be $x+5+c=12+c$ and if you rewrite the addition problem as a subtraction problem you will get $x=12+c-5-c$, which is the same as $x=12-5$ and can be written as an addition problem, $x+5=12$. In both cases the solution is $x=7$.

Student Activity Questions—Activity 1 (continued)

4. Solve each problem using arithmetic. Then show how you could set up an equation and use the file to solve the same problem.

a. Sondra received 18 text messages before noon on Monday. At the end of the day she had received a total of 37 text messages. How many text messages did she get the rest of the day?

Answer: $37-18=19$. An equation would be $x+18=37$. To solve this equation, you could subtract 18 from both sides of the original equation.

b. Sondra received 13 text messages after noon on Tuesday, and at the end of the day she had a total of 31 messages. How many text messages did Sondra get in the morning?

Answer: $31-13=18$. An equation would be $x+13=31$. To solve, you could subtract 13 from both sides of the original equation.

c. After Sondra deleted 13 of the text messages she received on Wednesday, she had 25 messages left for that day. How many did she have to start?

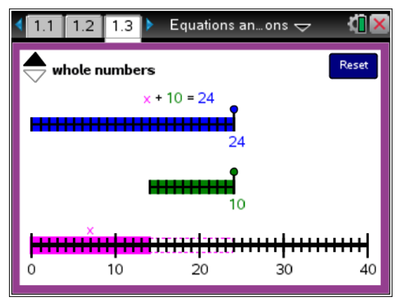
Answer: $25+13=38$. An equation would be $x-13=25$. To solve, you could add 13 to both sides of the original equation.

Part 2, Page 1.3

Focus: Students continue their investigation of solution-preserving moves with questions that involve fractions and decimals.

While continuing to work on page 1.3 students expand upon their exploration of solution-preserving moves.

Note: this part could be omitted if the focus is on whole numbers.



Class Discussion

The following questions relate to the same solution-preserving moves but involve fractions and decimals.

Reset the page. Use the arrow keys to select fractions.

- ***What is the solution to the equation? How do you know your answer is correct?***

  Answer: $\frac{5}{3}$ is the solution because $\frac{5}{3}+\frac{2}{3}=\frac{7}{3}$ is a true statement.

- ***Change D to 4. What do you notice about the solution?***

  Answer: The solution is $\frac{5}{4}$; the denominator has changed to 4.

Class Discussion (continued)

- ***Make a conjecture about what you think will be the solution for D =5? D = 6? Explain why you think your conjecture might be correct. Use the file to check your thinking.***

Answers may vary. The solutions for the equations will be $\frac{5}{5}$ and $\frac{5}{6}$. Any unit fraction—$\frac{1}{5}$, $\frac{1}{6}$, etc.—will always need 5 copies of that unit fraction added to 2 copies of the unit fraction to get 7 copies of the unit fraction.

Student Activity Questions—Activity 2

1. Reset the page so the equation is $x+\frac{2}{3}=\frac{7}{3}$. Drag the green segment to $\frac{0}{3}$.

a. Describe the equation.

Answer: $x+\frac{0}{3}=\frac{5}{3}$

b. What value was subtracted from both sides of the original equation to obtain the new equation?

Answer: $\frac{2}{3}$

c. Tami said that the new equation has the same solution as $x=\frac{5}{3}$. Do you agree with her? Why or why not.

Answer: Tami is right because $\frac{0}{3}=0$ and $x+0=\frac{5}{3}$ is the same as $x=\frac{5}{3}$.

2. Reset the page. Create the equation $x-\frac{3}{2}=\frac{7}{4}$ by dragging the green segment first and then the blue segment.

a. What is the solution to the equation?

Answer: $x=\frac{13}{14}$ or $x=3\frac{1}{4}$

b. Explain how you could find the solution by dragging the green segment.

Answer: Drag the green segment to $\frac{0}{4}$. This will give you the equation $x+\frac{0}{4}=\frac{13}{4}$ or $x=3\frac{1}{4}$.

c. What number was added to both sides of the original equation to make the new equation?

Answer: $\frac{3}{2}$ or $\frac{6}{4}$.

Student Activity Questions—Activity 2 (continued)

3. **Reset the page. Use the arrow to select decimals.**

   a. **Give at least two ways you could find the solution to the equation.**

   Answers may vary. Students might suggest: use the scale to read the value of *x* as 1.4; drag the green segment so it was 0 and get the equation $x+0=1.4$; subtract 1 from both sides of the equation.

   b. **Change the constant on the left side of the equation to 2. Predict what you think the solution will be. Explain your reasoning. Use the file to check your answer.**

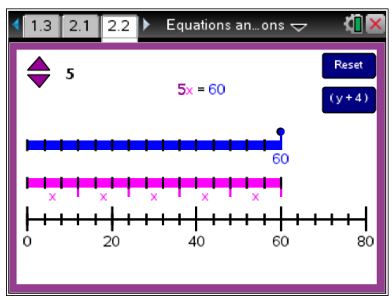
   Reasons may vary: $x=0.4$. Some might say that since $1.4+1=2.4$, and $1.4=0.4+1$, then $0.4+1+1=0.4+2=2.4$.

   c. **Reset the page. Change the constant on the right side of the equation to 3.4. Predict what you think the solution will be. Explain your reasoning. Use the file to check your answer.**

   Reasons may vary. Some might say that since $1.4+1=2.4$, you would need to have 1 more on the left side to make 3.4, so $x=2.4$.

Part 3, Page 2.2

Focus: This section introduces students to the solution-preserving move of multiplying both sides of an equation by the same nonzero number.



On page 2.2, dragging the handle on the blue segment changes the value of the equation.

Change the multiplier by using the up and down arrows.

To replace *x* with $y+4$, select the $y+4$ button.

To change the constant within the parenthesis drag the green handle or use the left and right arrows.

TI-Nspire Technology Tips

enter inserts or removes the value $y+4$ in the equation.

tab toggles between the blue and green segments.

ctrl del resets the page.

Class Discussion

The following questions focus on the equations of the form $ax=b$.

• ***What is the value of x on page 2.2? Is it a solution to the equation? How do you know?***	Answer: $x=12$ because $5(12)=60$ is true.
• ***Change the multiplier of x to 3. Describe what happened to the equation, the blue segment and each pink segment.***	Answer: The equation changed to $3x=36$. The blue segment changed to 36, and the length of the pink segment did not change.

Class Discussion (continued)

- ***What is the solution to the new equation?*** Answer: $x = 12$

Make a conjecture to answer the following questions. Then use the file to check your thinking.

How will the new equation compare to the old equation and how will the solutions to the original and new equations compare if you change the multiplier to

- ***4*** Answer: The equation will change to $4x = 48$, and the solution will remain the same, $x = 12$.
- ***1*** Answer: The equation will change to $1x = 12$, and the solution will remain the same, $x = 12$.
- $\frac{1}{3}$ Answer: The equation will change to $\frac{1}{3}x = 4$, and the solution will remain the same, $x = 12$.

Show how you could solve each problem using arithmetic and then show how you could use an algebraic equation and the file to find the solution.

- ***If Sallee had $70 to spend, how many $5 discount movie tickets could she buy?*** Answers may vary. Divide 70 by 5 to get 14 tickets; If $x =$ the number of movie tickets, the equation is $5x = 70$. To solve, multiply both sides by $\frac{1}{5}$ to get $x = 17$.
- ***Steve got $\frac{1}{4}$ of the batch of brownies his team made at school. If he had 19 brownies, how many brownies did they make all together?*** Answers may vary. Multiply 19 by 4 to get 76. If $x =$ the number of brownies they made all together, the equation is $\frac{1}{4}x = 19$. To solve, multiply both sides by 4.

Teacher Tip: Be sure students explicitly state what *x* represents in in the previous two questions. One example of confusion might be in the question above where *x* could represent the number of brownies Steve got or the total number of brownies his team made.

Student Activity Questions—Activity 3

1. **Create the equation $6x = 24$. Use the file to decide whether the new equations will have the same solution as $6x = 24$.**

   a. $\mathbf{12x = 48}$

   b. $\mathbf{10x = 28}$

   c. $\mathbf{\frac{1}{2}x = 8}$

   d. $\mathbf{3x = 12}$

   e. $\mathbf{1x = 4}$

   Answer: a., d. and e. have the same solution, $x = 4$.

2. a. **Describe how the equations in question 1 could be made from the original equation $6x = 24$.**

   Answers: a. multiply both sides by 2; b. (answers may vary) add 4*x* to the left side and 4 to the right side c. (answers may vary) multiply the left side by $\frac{1}{6}$ to get 1*x* and then by $\frac{1}{2}$, but the right side would have to be multiplied by $\frac{1}{3}$; d. multiply both sides by $\frac{1}{2}$; e. multiply both sides by $\frac{1}{6}$.

   b. **Which of the strategies you described in part 2a preserved the solution from the original equation?**

   Answer: multiplying both sides by the same number as in 1a, 1d, and 1e.

> **Teacher Tip:** In question 3, students solve a contextual problem using arithmetic and then create an equation that could be used to solve the problem to give them a sense of how the representation in symbols relates to the arithmetic.

3. **Do you agree or disagree with the following students. Explain why or why not.**

   a. **Teena says that you can multiply both sides of an equation by any number, and the new equation will have the same solution as the original equation. Do you agree with her? Why or why not?**

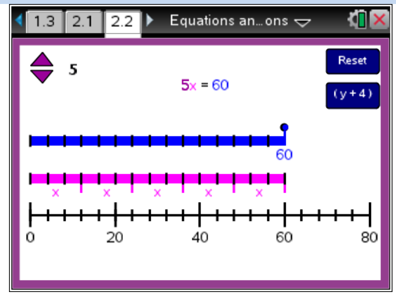
   Answer: Teena is wrong. You cannot multiply by 0 or you will get a statement that says $0 = 0$, without even an *x*. The solution to $5x = 20$ is $x = 4$; but $0x = 0(20)$ makes $0 = 0$, which is a true statement for any *x*.

   b. **Kurt says that every equation with a variable has some number that will be a solution.**

   Answer: Kurt is wrong because $0x = 10$ will never have a solution.

Part 4, Page 2.2

Focus: Students continue their investigation of solution-preserving moves with questions that involve fractions and decimals.



Teacher Tip: The emphasis in this section is not on distributing and creating an equation of the form $ax+b=c$ but rather on continuing to think about multiplying both sides of an equation as a solution-preserving strategy even if one of the sides consists of more than one value or term. This work should help students recognize in later work that multiplying only the 2 and the 12 by $\frac{1}{2}$ in the equation $2x+7=12$ is not a solution-preserving move.

Class Discussion

Have students…	**Look for/Listen for…**
Reset. Select $y+4$.	
• ***Describe what happened to each segment.***	Answer: The blue segment remained 60. Each pink segment that was *x* is now a pink segment labeled *y* and a green segment that is 4 units long.
• ***How many green segments are represented? What is the length of all of the green segments?***	Answer: There are five green segments, each 4 units long for a total length of 20.
• ***What is the solution?***	Answer: $y=8$
• ***Compare the solution for $5(y+4)=60$ to the solution to the original equation $5x=60$. What might explain any difference?***	Answer: The original solution was $x=12$; the solution to the equation $5(y+4)=60$ is $y=8$. But $y+4$ will be $8+4$ and will still equal 12.
For each of the following, write the equation, then make a conjecture about the value of y. Use the file to check your thinking. The green segment is	
• ***6***	Answer: $5(y+6)=60$; $y=6$, since $6+6=12$
• ***0***	Answer: $5y=60$, $y=12$, since $12+0=12$
• ***12***	Answer: $5(y+12)=60$; $y=0$, since $0+12=12$

Class Discussion (continued)

Reset the page. Change the equation to $4(y+8)=48$ by changing the length of the green segments.	
• ***What is the value of $y+8$? Explain your reasoning.***	Answer: $y+8=12$ because 4 times 12 is 48.
• ***What is the solution for y?***	Answer: $y=4$
• ***How many 8's are displayed on the number line? Explain why.***	Answer: Four 8's are on the number line because the equation has four *y*'s and four 8's.
• ***Change the multiplier to 2. How many 8's and how many y's are on the number line? Does the solution change? Why or why not?***	Answer: There are two 8's and two *y*'s in the graph. The solution does not change; it is still $y=4$ because the value of $y+8$ still has to be 12 in order for the product of 2 and $y+8$ to be 24.
For each of the following, change the equation $13(y+3)=65$ as described. Predict what will happen to the solution in each case. Explain your reasoning. Then use the file to check your thinking. What solution preserving moves could you make to create the new equation from the original?	
• ***Change the multiplier to 5.***	Answer: The solution will be $y=2$ because $5(y+3)=25$ so $y+3=5$ and $y=2$. Students may use different solution preserving moves; one might be divide both sides of the original equation by 13, then multiply both sides of the new equation by 5.
• ***Change the multiplier to 1.***	Answer: The solution will be $y=2$ because the equation will be $1(y+3)=5$ and $1(2+3)=5$ $1(2+3)=5$. One solution preserving move might be to divide both sides of the original equation by 13.
• ***Return to $13(y+3)=65$, then change the 3 to a 5.***	Answer: The solution will be $y=0$ because $13(0+5)=13(5)=65$.
• ***Return to $13(y+3)=65$, then change the 65 to 52.***	Answer: The solution will be $y=1$ because $13\times 4=52$ and $13(1+3)=52$.

Class Discussion (continued)

The following question is a summary question that could be assigned as a way to assess student understanding of the concepts covered in the three sections.

Write a paragraph explaining to someone in your home what a solution preserving move is with respect to equations. Describe the person you are writing for. Think of at least one question they might ask and how you would answer. Give several examples to support your explanation.

Answers will vary. Paragraphs should address multiplication and/ or division (but not by 0) and addition and subtraction. Possible questions might be: *Will your strategies work with any kinds of numbers or can you do more than one thing—like add and multiply?*

Student Activity Questions—Activity 4

1. Reset the page. Change the equation to $\left(\frac{1}{2}\right)(y+12)=18$.

a. Explain how the values associated with the lengths of the pink, green and blue segments are related.

Answer: The blue segment represents the end result of 18; the green segment is of length 12, and the pink segment is of length 24 for a total length of 36. Half of this length is 18.

b. What is the solution?

Answer: $y=24$

c. Predict the new equation and the solution to the new equation if the multiplier is changed to $\frac{1}{3}$. Check your prediction using the file.

Answer: The new equation will be $\left(\frac{1}{3}\right)(y+12)=12$ and the solution will still be $y=24$.

2. Decide whether the following statements are true or false. Use an example from the file to support your thinking.

a. Multiplying both sides of an equation by the same number will preserve the solution to the original equation.

Answer: True as long as the number you multiply by is not zero. Examples will vary.

b. The equation $3(x+4)=72$ will have the same solution as $3x+4=72$.

Answer: False because $3(x+4)$ has 3 x's and 3 fours not just one four. The solution to $3(x+4)=72$ will be $x=20$ and $3(20)+4=64$ not 72, so 20 is not a solution for $3x+4=72$.

c. The equation $x+27=39$ will have the same solution as $2x+27=78$.

Answer: False because the 27 has to be doubled as well as the *x* and the 39. The solution to $x+27=39$ is 12 but $2(12)+27=51$ not 78 so 12 is not a solution for $2x+27=78$.

Student Activity Questions—Activity 4 (continued)

d. **Multiplying both sides of an equation by a unit fraction is the same as dividing both sides of the equation by the denominator of the fraction.**

Answer: True because $\frac{1}{2}(a)$ is the same as $\frac{a}{2}$ which is the same as dividing a by 2.

e. **An equation can never have 0 as a solution.**

Answer: False because $4(y+15)=60$ has $y=0$ as a solution.

Deeper Dive — Page 1.3

Sam had $2.80 and was trying to decide which piece of fruit to buy. He could spend $0.40 for a banana, $1.30 for a pomegranate, or $1.90 for a mango.

- ***Describe how you would use arithmetic to find the amount of change in each case.***

  Answer: $2.80 – $0.40 = $2.40 in change; $2.80 – $1.00 = $1.50 in change; $2.80 – $1.90 = $0.90 in change.

- ***Describe how you could use the file to set up an equation involving addition and how you might find the solution.***

  Answers may vary: For the banana, *x* + $0.40 = $2.80; one strategy is to subtract $0.40 from both sides. For the pomegranate, *x* + $1.30 = $2.80; a strategy is to rewrite the addition problem as a subtraction, *x* = $2.80 – $1.30. For the mango, *x* + $1.90 = $2.80; a strategy might be to think about $1.90 as $1.80 + $0.10 and *x* + $1.90 = $2.80 would be the same as *x* + $1.80 + $0.10 = $2.80, so *x* + $0.10 = $1.00 and *x* would have to be $0.90.

Describe a mathematical operation that will change $12x=60$ into each of the following. (You might want to use the file to check your thinking.)

- $6x=60$

  Answer: Multiply both 12 and 60 by $\frac{1}{2}$ or divide both by 2.

- $4x=20$

  Answer: Multiply both 12 and 60 by $\frac{1}{3}$ or divide both by 3.

Deeper Dive — Page 1.3 (continued)

- ***$15x = 75$***

Answers may vary. Multiply both 12 and 60 by $\frac{5}{4}$ or $\frac{15}{12}$; divide both 12 and 60 by 12 and then multiply by 15; multiply both numbers by 15, then divide by 12.

- ***Find the solutions for the three equations above.***

Answer: The solutions are all $x = 5$.

Which of the following equations will have the same solution as $\left(\frac{1}{2}\right)(y+12) = 18$? Explain your reasoning.

a. $2(y+12) = 36$

b. $2(y+12) = 72$

c. $1(y+12) = 36$

d. $\left(\frac{1}{4}\right)(y+12) = 9$

Answer: b, c and d have the solutions $y = 24$.

Lorre says that to change the equation $15(y+3) = 75$ into $10(y+3) = 50$ you should multiply the 15 and 75 by 10 and then divide the new numbers by 15.

- ***Do you agree with Lorre? Why or why not?***

Answer: She is correct. Her strategy will give $150(y+3) = 750$ and dividing by 15 will produce $12(y+3) = 50$.

- ***Kong argued that it would be easier to divide by 15 (or multiply by $\frac{1}{15}$) to get the number 1 as a multiplier, then multiply by 10. Do you agree with Kong? Why or why not?***

Answer: I think Kong is right because dividing by 15 reduces the numbers in the equation to $(y+3) = 5$ and then multiplying by 10 is easy.

Deeper Dive — Page 2.2

- ***Ana thought it would be easier to divide everything by 3 first to get $5(y+1)=25$, and then it is easy to multiply by 2, but you would get $10(y+2)=50$ not $10(y+3)=50$. Do you agree with Ana? Why or why not?***

Answer: Ana is not correct because the multiplier applies to the whole quantity $(y+3)$ and not just to the numbers in the quantity like the 3. This is not a solution preserving move because using the file, in $15(y+3)=75$, there are five sets of $(y+3)$ and each $y=2$. The equation $5(y+1)=25$ has five sets of $(y+1)$, but in this case $y=4$. The new equations do not have the same solutions as the original.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. $\square + \frac{2}{3} = \frac{3}{4}$

   Enter a fraction that makes the equation true.

   SBA Practice test

   Answer: $\boldsymbol{\frac{1}{12}}$

2. Greg bought 4 notebooks for $6.40.

   a. Which equation can be used to determine the price, p, in dollars, of one notebook?

      i. $\frac{p}{4} = 6.40$ ii. $\frac{p}{6.40} = 4$ iii. $4p = 6.40$ iv. $6.40p = 4$

      Answer: iii) $\boldsymbol{4p = 6.40}$

   b. What is the price in dollars of one notebook?

      Answer: $1.60

   PAARC Practice Test

3. Chad stopped and filled the car with 11 gallons of gas. He had driven 308 miles with the previous 11 gallons of gas.

   a. How many miles per gallon did Chad's car get?

      Answer: $\boldsymbol{\frac{308}{11}}$ ***miles per gallon or 28 mph.***

   b. Chad's car continues to get the same number of miles per gallon. How many gallons of gas will Chad's car use to travel 672 miles?

      Answer: $\boldsymbol{\frac{672}{28}}$ ***or*** $\boldsymbol{\frac{168}{7}}$ ***or 24 gallons.***

   PAARC Practice Test

4. Which of the following equations does not have the same solution as the equation $n+18=23$?

   a. $23=n-18$

   b. $23=18+n$

   c. $18=23-n$

   d. $18+n=23$

   e. $n=23-18$

Adapted from NAEP, grade 8, 2011

Answer: a. $\boldsymbol{23=n-18}$

Student Activity Solutions

In these activities you will identify solution-preserving mathematical moves related to equations and moves that do not preserve solutions. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Adjust the length of the blue and green segments to create the equation $x+17=32$. Answer each of the following. Explain your reasoning in each case.

   a. If you make the green segment larger, what happens to the equation?

   Answer: The amount you increase the green segment is added to the right side of the equation as well as to the constant term on the left side.

   b. What happens to the pink segments and to the solution to the equation?

   Answer: The pink segments remain the same size, 15, and the solution, the value for x, is always the same, $x=15$.

   c. If you make the green segment smaller, what happens to the equation? To the solution to the equation?

   Answer: The amount is subtracted from the constant term on the left and from the 32. The solution stays the same, $x=15$.

2. Reset the page. Create the equation $x+17=32$ again.

   a. If you make the blue segment larger, what happens to the equation?

   Answer: The amount you increase the blue segment is added to the right side of the equation, and x increases in length by that same amount.

   b. What happens to the pink segments and to the solution to the equation?

   Answer: The pink segments increase in length. The solution, the value for x, gets larger as you increase the right side of the equation by the amount of that increase.

   c. If you make the blue segment smaller, what happens to the equation? To the solution to the equation?

   Answer: The amount is subtracted from the 32, and the rest of the equation stays the same; for example, when you make the blue segment 3 units shorter, the equation becomes $x+17=29$. The value for x, which is the solution, gets smaller by the same amount that the right side increased, $x=12$ for the example.

3. Students are discussing equations of the form $x+a=b$ where a and b are given numbers. Decide whether you agree or disagree with the statements some of them made. Give a reason for your answer.

   a. Sandra said that adding 2 to the left side of the equation and 3 to the right side of the equation will make a new equation whose solution is 1 more than the solution to the original equation.

   Answer: Sandra is right because you had one more on the right side so x would have to be one greater.

   b. Mark said that the solution to the equation will be $x=b-a$.

   Answers may vary. Mark is right because $x=b-a$ *makes a true statement:* $(b-a)+a=b$. *Or Mark is correct because an addition problem such as* $5+7=12$ *can be written as a subtraction problem in two different ways,* $5=12-7$ *or* $7=12-5$.

   c. Patou said that if you subtract 11 from both sides of an equation, the value of *x* for the new equation will be 11 less than the value of *x* in the original equation.

   Answer: Patou is not correct because if you subtract 11 from both sides, the amount you need to make the sides the same will not change.

   d. Mabel said that adding the same number to both sides of an equation will not change the solution.

   Answers may vary. Mabel is correct because if you have $x+5=12$ *and add a number c to both sides, the new equation will be* $x+5+c=12+c$ *and if you rewrite the addition problem as a subtraction problem you will get* $x=12+c-5-c$, *which is the same as* $x=12-5$ *and can be written as an addition problem,* $x+5=12$. *In both cases the solution is* $x=7$.

4. Solve each problem using arithmetic. Then show how you could set up an equation and use the file to solve the same problem.

   a. Sondra received 18 text messages before noon on Monday. At the end of the day she had received a total of 37 text messages. How many text messages did she get the rest of the day?

   Answer: $37-18=19$. *An equation would be* $x+18=37$. *To solve this equation, you could subtract 18 from both sides of the original equation.*

   b. Sondra received 13 text messages after noon on Tuesday, and at the end of the day she had a total of 31 messages. How many text messages did Sondra get in the morning?

   Answer: $31-13=18$. *An equation would be* $x+13=31$. *To solve, you could subtract 13 from both sides of the original equation.*

   c. After Sondra deleted 13 of the text messages she received on Wednesday, she had 25 messages left for that day. How many did she have to start?

   Answer: $25+13=38$. *An equation would be* $x-13=25$. *To solve, you could add 13 to both sides of the original equation.*

Activity 2 [Page 1.3]

1. Reset the page so the equation is $x+\frac{2}{3}=\frac{7}{3}$. Drag the green segment to $\frac{0}{3}$.

    a. Describe the equation.

    Answer: $x+\frac{0}{3}=\frac{5}{3}$

    b. What value was subtracted from both sides of the original equation to obtain the new equation?

    Answer: $\frac{2}{3}$

    c. Tami said that the new equation has the same solution as $x=\frac{5}{3}$. Do you agree with her? Why or why not.

    Answer: Tami is right because $\frac{0}{3}=0$ *and* $x+0=\frac{5}{3}$ *is the same as* $x=\frac{5}{3}$.

2. Reset the page. Create the equation $x-\frac{3}{2}=\frac{7}{4}$ by dragging the green segment first and then the blue segment.

    a. What is the solution to the equation?

    Answer: $x=\frac{13}{14}$ *or* $x=3\frac{1}{4}$

    b. Explain how you could find the solution by dragging the green segment.

    Answer: Drag the green segment to $\frac{0}{4}$. *This will give you the equation* $x+\frac{0}{4}=\frac{13}{4}$ *or* $x=3\frac{1}{4}$.

    c. What number was added to both sides of the original equation to make the new equation?

    Answer: $\frac{3}{2}$ *or* $\frac{6}{4}$.

3. Reset the page. Use the arrow to select decimals.

    a. Give at least two ways you could find the solution to the equation.

    Answers may vary. Students might suggest: use the scale to read the value of x as 1.4; drag the green segment so it was 0 and get the equation $x+0=1.4$*; subtract 1 from both sides of the equation.*

    b. Change the constant on the left side of the equation to 2. Predict what you think the solution will be. Explain your reasoning. Use the file to check your answer.

    Reasons may vary: $x=0.4$. *Some might say that since* $1.4+1=2.4$, *and* $1.4=0.4+1$, *then* $0.4+1+1=0.4+2=2.4$.

c. Reset the page. Change the constant on the right side of the equation to 3.4. Predict what you think the solution will be. Explain your reasoning. Use the file to check your answer.

Reasons may vary. Some might say that since $1.4+1=2.4$*, you would need to have 1 more on the left side to make 3.4, so* $x=2.4$*.*

Activity 3 [Page 2.2]

1. Create the equation $6x=24$. Use the file to decide whether the new equations will have the same solution as $6x=24$.

   a. $12x=48$

   b. $10x=28$

   c. $\frac{1}{2}x=8$

   d. $3x=12$

   e. $1x=4$

   Answer: a, d, and e have the same solution, $x=4$*.*

2. a. Describe how the equations in question 1 could be made from the original equation $6x=24$.

   Answers: a) multiply both sides by 2; b) (answers may vary) add 4x to the left side and 4 to the right side c) (answers may vary) multiply the left side by $\frac{1}{6}$ *to get 1x and then by* $\frac{1}{2}$*, but the right side would have to be multiplied by* $\frac{1}{3}$*; d) multiply both sides by* $\frac{1}{2}$*; e) multiply both sides by* $\frac{1}{6}$*.*

   b. Which of the strategies you described in part 2a preserved the solution from the original equation?

   Answer: multiplying both sides by the same number as in 1a, 1d, and 1e.

3. Do you agree or disagree with the following students. Explain why or why not.

   a. Teena says that you can multiply both sides of an equation by any number, and the new equation will have the same solution as the original equation. Do you agree with her? Why or why not?

   Answer: Teena is wrong. You cannot multiply by 0 or you will get a statement that says $0=0$*, without even an x. The solution to* $5x=20$ *is* $x=4$*; but* $0x=0(20)$ *makes* $0=0$*, which is a true statement for any x.*

   b. Kurt says that every equation with a variable has some number that will be a solution.

   Answer: Kurt is wrong because $0x=10$ *will never have a solution.*

Activity 4 [Page 2.2]

1. Reset the page. Change the equation to $\left(\frac{1}{2}\right)(y+12)=18$.

   a. Explain how the values associated with the lengths of the pink, green and blue segments are related.

   Answer: The blue segment represents the end result of 18; the green segment is of length 12, and the pink segment is of length 24 for a total length of 36. Half of this length is 18.

   b. What is the solution?

   Answer: $y=24$

   c. Predict the new equation and the solution to the new equation if the multiplier is changed to $\frac{1}{3}$.

   Check your prediction using the file.

   Answer: The new equation will be $\left(\frac{1}{3}\right)(y+12)=12$ *and the solution will still be* $y=24$.

2. Decide whether the following statements are true or false. Use an example from the file to support your thinking.

   a. Multiplying both sides of an equation by the same number will preserve the solution to the original equation.

   Answer: True as long as the number you multiply by is not zero. Examples will vary.

   b. The equation $3(x+4)=72$ will have the same solution as $3x+4=72$.

   Answer: False because $3(x+4)$ *has 3 x's and 3 fours not just one four. The solution to* $3(x+4)=72$ *will be* $x=20$ *and* $3(20)+4=64$ *not 72, so 20 is not a solution for* $3x+4=72$.

   c. The equation $x+27=39$ will have the same solution as $2x+27=78$.

   Answer: False because the 27 has to be doubled as well as the x and the 39. The solution to $x+27=39$ *is 12 but* $2(12)+27=51$ *not 78 so 12 is not a solution for* $2x+27=78$.

   d. Multiplying both sides of an equation by a unit fraction is the same as dividing both sides of the equation by the denominator of the fraction.

   Answer: True because $\frac{1}{2}(a)$ *is the same as* $\frac{a}{2}$ *which is the same as dividing a by 2.*

   e. An equation can never have 0 as a solution.

   Answer: False because $4(y+15)=60$ *has* $y=0$ *as a solution.*

