Building Concepts: Equations of the Form $ax + by = c$

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students investigate linear equations of the form $ax + by = c$. Students use coordinate grids to explore the “trade off” or “exchange” between the values of the two variables in order to maintain the constraint imposed by the constant *c*.

Develop linear combinations of two quantities and investigating what happens when a constraint is imposed on the linear combination $ax + by$ for a given *a* and *b*.

Learning Goals

1. Explain the meaning of a solution for a linear equation in two variables;
2. recognize that the solution set for a linear equation in two variables can be represented graphically as a straight line;
3. write two-variable linear equations to represent a situation involving a constraint;
4. explain the “exchange” relationship between two variables in contexts involving linear combinations and constraints;
5. identify solutions to a linear equation of the form $ax + by = c$.

Prerequisite Knowledge

Equations of the form $ax + by = c$ is the thirteenth lesson in a series of lessons that investigates the statistical process. In this lesson, students investigate linear equations of the form $ax + by = c$. Prior to working on this lesson, students should have completed *Linear Inequalities in One Variable* and *Solving Equations*. Students should understand:

- how to graph in a coordinate plane;
- how to find solution sets for linear equations in one variable;
- the concept of solution-preserving moves.

Vocabulary

- **linear equation:** an algebraic equation that makes a line when it is graphed
- **variables:** letters or symbols that stand for a changing quantity
- **constraint:** a restriction placed on a variable
- **solution:** the set containing value(s) of the variable(s) that satisfy all equations and/or inequalities
- **intercepts:** the coordinates of points at which lines and curves intersect a coordinate axis
- **linear combinations:** expressions constructed from a set of terms by multiplying each term by a constant and adding the results

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Equations of the form $ax + by = c$ _Student.pdf
- Equations of the form $ax + by = c$ _Student.doc
- Equations of the form $ax + by = c$ .tns
- Equations of the form $ax + by = c$ _Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

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TEACHER NOTES

Mathematical Background

Not all linear equations in two variables are written explicitly in the form $y = mx + b$, and in many contexts, doing so distorts the meaning. A more general form of a line is $ax + by = c$ because it includes vertical lines. In this context of a general linear equation in two variables, students investigate the meaning of a solution for the equation, i.e., the set of all possible values for *x* and for *y* that make the equation a true statement. It is important to note what each variable represents. In many contexts involving linear equations, there is not a clear independent/dependent relationship between the variables. Rather, the equation represents a statement about a linear combination, an expression constructed from a set of variables by multiplying each variable by a constant and adding the results (e.g., a linear combination of *x* and *y* would be any expression of the form $ax + by$, where *a* and *b* are constants), (the "trade off") if a given constraint, *c*, is to be maintained (resulting in $ax + by = c$).

In the case of an equation of the form $ax + by = c$, the solution set forms a straight line. Note that the words "the solution" are different than the words "a solution"; the first phrase implies the entire solution set, which in this case will be represented by the entire set of points on a line determined by the equation, and the second implies only one point or ordered pair of the entire set of possible solutions.

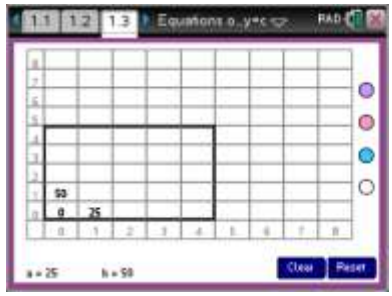
In this lesson, students use coordinate grids to explore the "trade off" or "exchange" between the values of the two variables in order to maintain the constraint imposed by the constant *c*. For example, if *k* represents the number of children's tickets and *a* the number of adult tickets, where the price of an adult ticket was $6 and of a children's ticket was $4, the cost of buying *a* many adult and *k* many children's tickets would be the linear combination, $6a + 4k$. If the total cost of the tickets was $48, the constraint of 48 would determine the equation $6a + 4k = 48$. Note that an increase in three children's tickets would decrease the possible number of adult tickets by two.

Students should be familiar with graphing in a coordinate plane and finding solution sets for linear equations in one variable.

The opening activity is adapted from Kindt, M., Abels, M., Meyer, M., Pligge, M. (1998). *Comparing Quantities*. From *Mathematics in Context*. Directed by Romberg, T. & deLange, J. Austin, TX: Holt, Rinehart, Winston.

Part 1, Page 1.3

Focus: Develop linear combinations of two quantities and investigating the result when a constraint is imposed on the linear combination $ax + by$ for a given *a* and *b*.



On page 1.3, students can use the arrow keys or the cursor to select a cell; they can enter values in the cells using the keypad.

Select a color or **menu> Highlight** and a color, select a cell, and **enter** to shade the cell.

Select **menu> Coefficients** and choose *a* or *b* to edit.

Undo erases the last step.

Clear displays a whole new grid.

Reset returns to the original grid.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the buttons.

enter highlights a selected cell.

ctrl del resets the page.

Class Discussion

***Suppose erasers sell for 25 cents and pencils for 50 cents*.**

- ***If you have $1.25, how many of each can you buy?***

  Answer: Students should guess and check to find they can buy 3 erasers and 1 pencil or 1 eraser and 2 pencils.

- ***Suze decided to make a table to help her think about the cost of pencils and another one for the erasers. Fill out the two tables*.**

Number of erasers	Cost
1	25
2	50
3	
4	
5	

Number of pencils	Cost
1	50
2	100
3	
4	
5	

Class Discussion (continued)

Answer:

Number of erasers	Cost
1	25
2	50
3	75
4	100
5	125

Number of pencils	Cost
1	50
2	100
3	150
4	200
5	250

- ***Jess thought of another way to make a table to help him keep track. He called it a combination table. What does the 75 represent?***

  Answer: The 75 represents the cost of 1 pencil and 1 eraser.

Pencils			
2			
1	**50**	**75**	
0		**25**	
	0	**1**	**2**

Erasers

Jess's combination table is on page 1.3. The numbers (0 to 8) in gray are the row numbers at the left and the column numbers at the bottom.

Answer: 3 rows or $3(2+3)$ will give you 6 circles and 9 squares.

- ***Fill in the columns labeled 0, 1, 2, and 3 starting at the bottom within the rectangle outlined on the grid (that is, up to but not including the row beginning with a 5). What does the value in the column corresponding to the 3 and the row corresponding to the 4 represent?***

  Answer: The 275 is the cost of three erasers (75 cents) and four pencils (200 cents) or $2.75.

- ***What do the numbers in the column corresponding to the 1 on the bottom row of the table represent?***

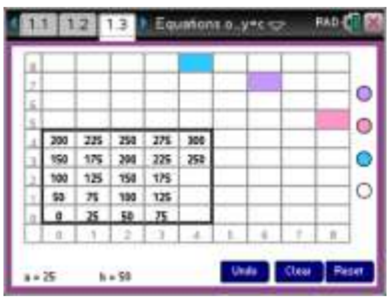
  Answer: The numbers represent the cost for 1 to 4 pencils and 1 eraser.

Class Discussion (continued)

- ***Enter the cost of four pencils and four erasers in the table. What patterns can you see in the values? Why do the patterns make sense?***

  Answers may vary. The value would be $3.00. Students might notice that if you go up one row, the value increases by 50 cents, which makes sense because you are increasing by one pencil each time; going to the right on column, the values increase by 25 cents because you are adding another eraser; going up the diagonal adds 75 cents because you are adding both a pencil and an eraser.

- ***Without filling in any other cells, highlight the cells as shown in the diagram below. Find at least two ways to figure out the value that goes in each cell.***



  Answers will vary. Have students show their strategies and color the pathways accordingly when students use patterns in the table.

  Possible strategies for the aqua cell: continue up from 300 by adding 50; multiply $4(25)+8(50)=500$ for four erasers and eight pencils.

  Possible strategies for the lavender cell: multiply $6(25)+7(50)=500$ for six erasers and seven pencils; fill in the diagonals 375, 450, until you are right below the lavender cell, then add 50; move up 50 at a time from 300 until you are in the same row as the lavender cell (450), then add 25 twice.

  Possible strategies for the pink cell: move to the right from the 300 adding 25 each time until you reach the edge, then go up one by adding 50 for 450; move two down from the lavender cell, subtracting 50 each time, then right two cells adding 25 each time.

Finish filling out the cells in the rectangular outline.

- ***Samee noticed that if he followed the diagonal from the 400 in the top left to the 200 in the bottom right, it went down 25 at a time and wondered why. What would you tell him?***

  Answer: You are moving down 1 which means you are decreasing the number of pencils so you subtract 50, but then you move to the right one, which means you added an eraser so you add 25 back for a net loss of 25 each time.

Class Discussion (continued)

• ***Sorin followed the diagonal starting at 50 in the lower left and going to 575. Explain to Sorin how the values on this diagonal change and why.***	Answer: You are moving to the right 1, which means you are adding an eraser (25) and up 1, which means you are adding a pencil (50) so you have added a total of 75 each time.
• ***Find another pattern in the table. Describe your pattern and explain why your pattern makes sense. Be ready to share your thinking with a partner.***	Answers will vary. One pattern that might come up is one where there is no change in the values; for example, over two and down one from any starting point results in the same value.
Suppose you only had $2.00 to spend on pencils and erasers.	
• ***Find the possible combinations of pencils and erasers you could buy. Highlight the combinations in a different color.***	Answer: 4 pencils and no erasers; 3 pencils and 2 erasers; 2 pencils and 4 erasers; 1 pencil and 6 erasers; and no pencils and 8 erasers.
• ***How do you know that you have them all?***	Answer: Because the number of pencils and number of erasers are whole numbers, there is no other possible combination possible that will cost $2.
• ***How are the cells in the table with 200 related?***	Answer: They are lined up. To get from one cell to the other, you go down 1 and to the right 2, which is the same as getting one less pencil and two more erasers, or up one and to the left 2, which is the same as getting one more pencil and two less erasers.
• ***Jason said that you could get another combination by "trading" in one pencil for two erasers. What would you say to Jason?***	Answer: Jason is correct, because losing a pencil takes off 50, but adding two erasers puts the 50 back, so you are still at $2.
Do not Reset the page.	
Suppose you had $3.00 to spend on pencils and erasers.	
• ***Could you buy just pencils? If so, how many?***	Answer: You could buy 6 pencils and no erasers for $3.
• ***Use the table to figure out the combinations of pencils and erasers you could buy for $3. Highlight the combinations that work in the table and explain how the table helps you find the combinations.***	Answer: You could buy 6 pencils and no erasers; 5 pencils and 2 erasers; 4 pencils and 4 erasers; 3 pencils and 6 erasers; and no pencils and 8 erasers. You can find the answers by looking at the cells that have 300.

Class Discussion (continued)

- ***Are any other combinations of pencils and erasers possible if the total cost is $3? Explain how you know.***

Answer: The table does not show all of the possible combinations, because you also could have 10 erasers and 1 pencil or 12 erasers and no pencils. You can see that the pattern in the table can keep going—exchanging 1 pencil for 2 erasers by going down one cell and to the right two cells off the table. You can do this twice.

- ***How are the combinations of pencils and erasers you could buy for $3 and the combinations you could buy for $2 related?***

Answer: The combinations make two "parallel" lines in the table.

The total amount of money you spend can be called a constraint.

- ***Decide whether the combination of 7 erasers and 3 pencils satisfies the constraint of $3.25. Explain how you know.***

Answer: Yes, because $30(50) + 7(25) = 150 + 175 = 325$.

- ***Describe two ways you could get a combination that had more erasers and fewer pencils or more pencils and fewer erasers from the combination in the question above and still spend $3.25.***

Answer: You could trade 1 pencil for 2 erasers to have 9 erasers and 2 pencils, or you could trade 2 erasers for 1 pencil to get 5 erasers and 4 pencils.

- ***What is the maximum number of pencils you can buy for $3.25? Explain how you know it is the maximum number.***

Answer: The maximum number of pencils is when there is only 1 eraser and 6 pencils because 6 pencils will cost $3 and 7 pencils will cost $3.50. To spend $3.25, you have to buy an eraser also. It is the maximum number of pencils, because you cannot buy half a pencil or a negative number of pencils.

- ***Using the maximum number of pencils you can buy from the question above, give the other possible combinations. Describe two different ways in which you could find the combinations.***

Answers may vary. You could find the other $3.25s on the table; you could continue to exchange one pencil for two erasers to get 3 erasers and 5 pencils; 5 erasers and 4 pencils; 7 erasers and 3 pencils; 9 erasers and 2 pencils; 11 erasers and 1 pencil; 13 erasers and no pencils.

Student Activity Questions—Activity 1

1. **Suppose Evangeline is at the arcade. The bowling game costs 2 tokens, while the asteroid game costs 3 tokens. Let *a* be the number of tokens it takes to play the asteroid game and *b* the number of tokens it takes to play the bowling game.**

   a. **Change *a* and *b* on page 1.3 to represent the number of tokens for bowling games and asteroid games. Predict the following patterns: moving up a column; moving across a row to the right; moving up the diagonal from the cell in the lower left to the cell in the upper right. Fill in the values of the table to check your predictions.**

   Answers: Moving up the cells in the columns increases each value in the cell by 2, while moving to the right across the cells in the rows increases the value in each cell by 3. When moving up the diagonal, each cell increases by 5.

   b. **Suppose Evangeline had 24 tokens to spend. What combinations of games could she play?**

   Answers: 0 bowling games, 8 asteroid games; 3 bowling games and 6 asteroid games; 6 bowling games and 4 asteroid games; 9 bowling games and 2 asteroid games; and 12 bowling games and no asteroid games. (Note that the last two solutions are not displayed in the table.)

   c. **What "exchange" of games will create all the possible combinations of asteroid games and bowling games that cost the same number of tokens? Explain your thinking.**

   Answer: Two bowling games can be exchanged for three asteroid games because the number of tokens for two bowling games is equal to the number of tokens for three asteroid games.

   d. **Marj accidentally made her table with *a* and *b* reversed; *b* as the number of asteroid games and *a* as the number of bowling games. Would her answers for question 1b be different? Why or why not?**

   Answer: No, the number of combinations would be the same for the games; the values in the table would be in a different place but would come from the same numbers. The diagonal would have the same values.

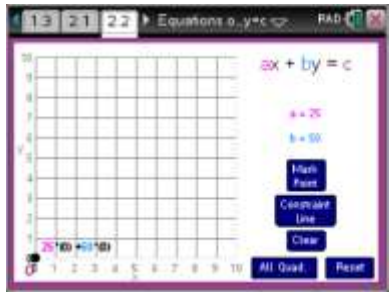
Part 2, Page 2.2

Focus: Develop understanding that the solution to an equation of the form $ax + by = c$ will be multiple values of *x* and *y* (an infinite number of ordered pairs) that make the equation true.

On page 2.2, students can use the arrow keys to move the point; they can use **Enter** or **Mark Point** to select the point.

Select **menu> Coefficients** and choose *a* or *b* to edit.

Note that the grid can be extended to all four quadrants, but that extension is not a part of this lesson.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the buttons and *a* and *b*.

Arrow keys move the selected board.

ctrl del resets the page.

Class Discussion

This page returns to the situation where an eraser costs $0.25 and a pencil costs $0.50.

- ***Move the dot from (0, 0) to the right six units and up four units. Mark the point. What does the number under that point represent?***

  Answer: The new point is (6, 4), and the number represents the cost of 6 erasers and 4 pencils, $3.50.

- ***Find two more points that would show a combination of pencils and erasers that would cost $3.50. Explain how you found the points.***

  Answers will vary. Two more points might be (8 erasers,3 pencils) and (10 erasers, 2 pencils). You can find these points by exchanging one pencil for 2 erasers, starting from the point (6, 4). (Some students might find the response by dragging to find another 350.)

- ***Consider the equation*** $25x + 50y = 350$.

  How are the points you found in the question above related to this equation? Explain your reasoning.

  Answer: The points make the equation a true statement because $25(8) + 50(3) = 200 + 150 = 350$; $25(10) + 50(2) = 250 + 100 = 350$; $25(4) + 50(5) = 350$.

- ***In the equation, what does each of the following represent: x; y; 25x; 50y?***

  Answer: The *x* represents the number of erasers; *y* represents the number of pencils; 25*x* represents the cost of *x* erasers and 50*y* represents the cost of *y* pencils.

Class Discussion (continued)

Remember that a solution to an equation is a number that makes the equation true when substituted for the variable. When the equation has two variables, a solution will be a pair of numbers representing values for each variable that makes the equation true.

- ***Write down three ordered pairs (x, y) that represent solutions to the equation $25x + 50y = 350$. Explain how you know that you have a solution.***

  Answers may vary. Possible solutions could be (8, 3); (6, 4) and (4, 5). I know they are solutions, because as shown in Student Activity 1, they make the equation a true statement.

- ***Find a solution to the equation $25x + 50y = 350$ when $y = 0$. Write your response as ordered pair. Interpret the ordered pair in terms of buying pencils and erasers.***

  Answer: (14, 0) represents buying 14 erasers and no pencils for a cost of $3.50.

- ***Find a solution to the equation when $x = 0$. Describe two ways you could find an answer.***

  Answer: You can extend the pattern by going from (4, 5) exchanging two erasers for one pencil twice to get (0, 7) or you could replace the *x* in the equation by 0 to get and equation you can solve to find the value of *y:* $25(0) + 50y = 350$ so $50y = 350$ and $y = 7$.

- ***If the values of x and y have to be non-negative whole numbers, list all of the solutions to the equation $25x + 50y = 350$ in a table. (Use the points you plotted in the questions above to help fill in the table.) Mark as many points as you can on the graph. What relationship exists among the points in the table?***

  Note: Have students build their own table so they are responsible for determining the number of solutions.

  Answer

x	*Y*
0	14
1	12
2	10
3	8
4	6
5	4
6	2
7	0

  The points lie on a straight line.

Class Discussion (continued)

Given the equation $25x + 50y = 450$,

- ***Mark all of the solutions to the equation you can see in the plot. How many solutions have you marked?***

  Answer: Six solutions can be displayed in the grid.

- ***Have you marked all possible solutions, given that both x and y must be whole numbers? How do you know?***

  Answer: No, some of the solutions will be off the grid to the lower right because you can exchange a pencil for two erasers starting from (10, 4) to get (12, 3); (14, 2); (16, 1); and (18, 0).

- ***Marissa said because the equation*** $25x + 50y = 450$ ***has two different variables, x and y, the values of x and y have to be different numbers. Based on your work in this question, what would you say to Marissa?***

  Answer: She is wrong because one of the solutions is 6 pencils and 6 erasers, $25(6) + 50(6) = 450$.

- ***The value of x when*** $y = 0$ ***is called the x-intercept, and the value of y when*** $x = 0$ ***is called the y-intercept. Find the x- and y-intercepts for the equation*** $25x + 50y = 450$. ***How are the intercepts related to the graph?***

  Answer: The *x*-intercept is 18 (18 erasers) and is associated with a point on the *x*-axis. The *y*-intercept is 9 (9 pencils) and would be associated with a point on the *y*-axis if the grid went that far.

Steel cable sells for $12 a foot and copper cable for $18 a foot. Cable is sold in fractions of a foot. Let x equal the number of feet of steel cable and y equal the number of feet of copper cable.

- ***What is the maximum amount of steel cable you can buy for $120? Where is the associated point on the grid?***

  Answer: 10 feet. The point is on the horizontal axis at $x = 10$, $y = 0$.

- ***Find three different combinations of the amounts if steel and amount of copper cable Po could buy with $120. Write your answer in an ordered pair and interpret your answer in terms of the amount of steel and cable.***

  Answer: (7, 2) or 7 feet of steel cable and 2 feet of copper cable; (4, 4) or 4 feet of both steel and copper cable; (1, 6) or 1 foot of steel cable and 6 feet of copper cable.

- ***Suppose Po has to figure out the number of feet he can buy of each kind of cable with $120. What kind of "exchange" can he make between the amount of steel and copper cable and still stay with a total cost of $120? Give an example to show what you are thinking.***

  Answer: Po can trade in 3 feet of steel cable for 2 feet of copper cable. For example, if he has 7 feet of steel cable and 2 feet of copper, he can have 3 less feet of steel cable for 2 more feet of copper cable: 4 steel and 4 copper would make $12(4) + 18(4) = 48 + 72 = 120$.

Class Discussion (continued)

- ***Write an equation that expresses the total cost of $120 for* x *feet of steel cable and* y *feet of copper cable and explain what each of the part of the equation represents.***

Answer: $12x + 18y = 120$. If *x* represents the number of feet of steel cable, then

12*x* represents the total cost of *x* feet of steel cable. If *y* represents the number of feet of copper cable, then 18*y* represents the total cost of *y* feet of copper cable wire.

Po needs exactly 3 feet of copper cable, but he only has $120 total to spend.

- ***How much steel cable can he buy? Explain how you can use the graph to find the solution.***

Answer: If you look at the possibilities on the graph for 3 feet of copper cable but a constraint of 120, you can see that 5 feet of steel cable will cost $114 and 6 feet of steel cable will cost $126. So the answer has to be in between 5 and 6 feet of cable. Exactly halfway between 114 and 126 will be 120 (add 6 to 114 and take 6 away from 126). You need to add 6 to 114, but that is $\frac{1}{2}$ of the increase from 5 to 6 $\left(12 \times \left(5 + \frac{1}{2}\right)\right)$. So, the amount of steel cable he can buy will be $5\frac{1}{2}$ feet.

- ***How can you prove that your answer to the question above is correct?***

Answer: The computations will be $12\left(5\frac{1}{2}\right) + 18(3) = 66 + 54 = 120$.

- ***Mari says she could buy 3 feet of steel cable and then $4\frac{2}{3}$ feet of copper cable. Is she right? Why or why not?***

Answer: She is correct, because $12(3) + 18\left(4\frac{2}{3}\right) = 36 + 84 = 120$.

- ***Use the graph to find one other combination of lengths of steel and copper cable she could buy for $120 where at least one of the lengths is a fraction.***

Answers may vary. One possibility is halfway between two feet of steel cable and 3 feet of steel cable or $2\frac{1}{2}$ feet of steel cable and 5 feet of copper for $12\left(2\frac{1}{2}\right) + 5(18) = 30 + 90 = 120$.

Class Discussion (continued)

What would you say to each of the following students?

- ***Tessa says that the solutions to $12x + 18y = 120$ can be any positive number for x that is greater than 0 and a positive number for y that makes the equation true. So, for example, $12\left(\frac{1}{2}\right) + 18\left(\frac{19}{3}\right) = 120$ so $\left(\frac{1}{2}, \frac{19}{3}\right)$ is a solution for the equation.***

  Answer: Tessa is correct. Lots of different fraction combinations of *x* and *y* will make the equation true as long as *x* and *y* are both greater than 0.

- ***Sibron says that if you were just looking at the equation $12x + 18y = 120$ without thinking about the feet of cable, you could have $x = -8$ and $y = 12$ as a solution to the equation.***

  Answer: Sibron is correct, because $12(-8) + 18(12) = -96 + 216 = 120$.

- ***Grace says that all of the solutions for $12x + 18y = 120$ will work for the equation $2x + 3y = 20$.***

  Answer: Grace is right; for example, when $x = 4$ and $y = 4$, then $2(4) + 3(4) = 8 + 12 = 20$ but $12(4) + 18(4) = 48 + 72 = 120$. A solution preserving move is to multiply both sides of an equation by a common factor, in this case $\frac{1}{6}$.

- ***Lani conjectured that if you drew a line through the ordered pairs that were solutions to $2x + 3y = 20$ and extended the line past the first quadrant, all the points lying on the line would be in the solution set for the line.***

  Answers may vary. Lani is correct.

Student Activity Questions—Activity 2

1. **Let *x* represent the number of glasses of water and *y* represent the number of bottles of juice.**

   Water is free and juice costs $1.50. Set up the combination table for the combinations of the number of glasses of water and the number of bottles of juice you can get for $4.50.

   a. **What is the equation?**

   Answer: $0x + 150y = 450$ or $150y = 450$.

   b. **Mark three points that satisfy the constraint. Give the ordered pairs for those points.**

   Answer: (5, 3), which means 5 glasses of water and 3 bottles of juice; (1, 3), or a glass of water and 3 bottles of juice; (4, 3) or 4 glasses of water and 3 bottles of juice.

Student Activity Questions—Activity 2 (continued)

c. **Describe all of the solutions for the equation and the corresponding graph of the points that are solutions.**

Answer: The solution will be any whole number of glasses of water—or even fractions, but always 3 bottles of juice. The graph will be a horizontal line going through all of the points where the *y*-value is 3.

d. **Create a context for an equation where the points in the solution for the equation would graph as a vertical line.**

Answers may vary. One possible answer is to make the *x* represent the number of bottles of juice and *y* the number of glasses of water. Then all of the ordered pairs (3, 1), (3, 3) and so on would lie on a vertical line.

2. **Decide whether the following are sometimes true, always true, or never true. Be ready to explain your reasoning.**

a. **The expression $4x + 3y$ will have the same value as long as an increase of 3 in the value of *x* is offset by a decrease of 4 in the value of *y*.**

Answer: Always true, because when the exchange between 4 and 3 is maintained, the total value will be constant.

b. **If you change $4x + 3y = c$ to $4x + 3y = d$ where *d* is different than *c*, the line graphs of the solution sets will be parallel.**

Answer: Always true because in both cases an increase of 3 in the value of *x* is offset by a decrease of 4 in the value of *y* and going the other way, a decrease of 3 the value of *x* is offset by an increase of 4 in the value of *y*.

c. **If *a* is not zero and you solve an equation of the form $ax = b$ or $x + a = b$ for *x*, you will have exactly one solution.**

Answer: True, because only one point will make the equation true; other points will either make the equation more than or less than *b*.

d. **If *a* and *b* are not zero, then an equation of form $ax + by = c$ will have exactly one solution for *x* and *y*.**

Answer: Never true, because the solution will be a set of points on a line; for example, (6, 3) will be one a point on the line that makes the equation true but so will the point (9, 1).

e. **The solution to an equation of the form $2x + 5y = c$ is a point on a line.**

Answer: Never true because the solution will be the entire set of points on a line.

Deeper Dive

Consider the equation of the form $ax + by = c$, where $a = 0$, $b = 8$ and $c = 12$.

- ***Find several points that would make the equation true.***

  Answer: The equation would become **=** $0x + 8y = 12$ or $8y = 12$, and any point that has a *y*-value = $\frac{3}{2}$ such as $\left(2\frac{3}{2}\right)$ or $\left(5,\frac{3}{2}\right)$ would work.

- ***Describe the solution set in general.***

  Answer: The solution set would be all of the points on a horizontal line where all of the *y*-values are $\frac{3}{2}$.

- ***Suppose that b = 0 and the equation were of the form $8x + 0y = 12$. Find some elements of the solution set.***

  Answer: Points of the form $\left(\frac{3}{2}, 5\right)$ or $\left(\frac{3}{2}, 2\right)$ would be in the solution set.

- ***How does the solution set to the question above compare to the solution set for the first question?***

  Answer: The solution set for c) will have a $\frac{3}{2}$ as the first element in the ordered pairs in the solution, while the ones for a) will have the $\frac{3}{2}$ as the second element in the ordered pairs in the solution. The solutions for the two problems will form perpendicular lines intersecting at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$ with the solution for c) the question above a vertical line with all of the *x*-coordinates $\frac{3}{2}$ and the solution for the first question a horizontal line with all of the *y*-coordinates $\frac{3}{2}$.

Given $ax + by = c$, find values for* a*,* b*, and* c *that are positive whole numbers so that the only solutions with integer coordinates are the intercepts.

Answer: Any two numbers without a common factor will work for *a* and *b*; where *c* is the product of *a* and *b*; for example $a = 2$, $b = 3$ and $c = 6$ will produce $2x + 3y = 6$, where (3, 0) and (0, 2*)* are the only integer solutions because if *x* was larger than 3, 2*x* would be larger than 6 and *y* would have to be negative to bring the sum to 6; the only whole number less than 2 is 1 and if $x = 1$, $y = \frac{4}{3}$ to make the sum equal 6.

Deeper Dive (continued)

Find the value of x in the marked cell in each combination table.

x					
		15			
0	**6**				

Answer: $x = 12$

	48				
	38				
0			x		

Answer: $x = 24$

Deeper Dive (continued)

		26		**X**	
			21		
0					

Answer: $x = 34$

				X	
	45				
			65		
0					

Answer: $x = 110$

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which of the following ordered pairs (x, y) is a solution to the equation $2x + 3y = 6$?

   a. (6, 3)

   b. (3, 0)

   c. (3, 2)

   d. (2, 3)

   e. (0, 3)

adapted from NAEP, Grade 8 1996

Answer: b

2. The point (4, k) is a solution to the equation $3x + 2y = 12$. What is the value of k?

   a. 3

   b. 0

   c. 2

   d. 3

   e. 4

NAEP 2011, Grade 8

Answer b

3. Consider this equation; $c = ax + by$. Joseph claims that if a, b, and c are non-negative integers, the equation has exactly one solution for x. Select all cases that show Joseph's claim is incorrect.

   a. $a - b = 1$, $c = 0$

   b. $a = b$, $c \neq 0$

   c. $a = b$, $c = 0$

   d. $a \neq b$, $c = 0$

   e. $b = 0$, $a = c$

Adapted from PARCC Math Spring Operational Grade 8 End of Year release item VF 646143

Answer e. gives an example of a case where there is only 1 solution; all of the others have an infinite number of solutions. a. to d. show Joseph's claim to be incorrect because a. $2x + y = 0$; for b. $2x + 2y = 4$; c. $2x + 2y = 0$; for d. $3x + 4y = 0$; e. $3 = 3x$.

4. The admission price to a movie theater is $7.50 for each adult and $4.75 for each child. Which of the following equations can be used to determine *T*, the total admission price, in dollars, for *x* adults and *y* children?

   a. $T = (7.50x + 4.75y)(x + y)$

   b. $T = 7.50x + 4.75y$

   c. $T = 7.50y + 4.75x$

   d. $T = (7.50x)(4.75y)$

   e. $(7.50 + 4.75) + (x + y)$

NAEP Grade 8, 2011

Answer: b

5. Bags of Healthy Snack Mix are packed into small and large cartons. The small cartons contain 12 bags each. The large cartons contain 18 bags each.

   Meg claimed that she packed a total of 150 bags of Healthy Snack Mix into 2 small cartons and 7 large cartons.

   Could Meg have packed the cartons the way she claimed?

NAEP Grade 8, 2011

Answer: Yes, because if* s *is the number of small cartons and* b *is the number of large cartons,* $12s + 18b = 150$ *is true for* $s = 2$ *and* $b = 7$*.

Student Activity Solutions

In these activities, you will identify solutions to a linear equation of the form $ax + by = c$. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Suppose Evangeline is at the arcade. The bowling game costs 2 tokens, while the asteroid game costs 3 tokens. Let *a* be the number of tokens it takes to play the asteroid game and *b* the number of tokens it takes to play the bowling game.

   a. Change *a* and *b* on Page 1.3 to represent the number of tokens for bowling games and asteroid games. Predict the following patterns: moving up a column; moving across a row to the right; moving up the diagonal from the cell in the lower left to the cell in the upper right. Fill in the values of the table to check your predictions.

   Answers: Moving up the cells in the columns increases each value in the cell by 2, while moving to the right across the cells in the rows increases the value in each cell by 3. When moving up the diagonal, each cell increases by 5.

   b. Suppose Evangeline had 24 tokens to spend. What combinations of games could she play?

   Answers: 0 bowling games, 8 asteroid games; 3 bowling games and 6 asteroid games; 6 bowling games and 4 asteroid games; 9 bowling games and 2 asteroid games; and 12 bowling games and no asteroid games. (Note that the last two solutions are not displayed in the table.)

   c. What "exchange" of games will create all the possible combinations of asteroid games and bowling games that cost the same number of tokens? Explain your thinking.

   Answer: two bowling games can be exchanged for three asteroid games because the number of tokens for two bowling games is equal to the number of tokens for three asteroid games.

   d. Marj accidentally made her table with *a* and *b* reversed; *b* as the number of asteroid games and *a* as the number of bowling games. Would her answers for question 1b be different? Why or why not?

   Answer: No, the number of combinations would be the same for the games; the values in the table would be in a different place but would come from the same numbers. The diagonal would have the same values.

Activity 2 [Page 2.2]

1. Let *x* represent the number of glasses of water and *y* represent the number of bottles of juice.

   Water is free and juice costs $1.50. Set up the combination table for the combinations of the number of glasses of water and the number of bottles of juice you can get for $4.50.

   a. What is the equation?

   Answer: $0x + 150y = 450$ *or* $150y = 450$.

b. Mark three points that satisfy the constraint. Give the ordered pairs for those points.

Answer: (5, 3), which means 5 glasses of water and 3 bottles of juice; (1, 3), or a glass of water and 3 bottles of juice; (4, 3) or 4 glasses of water and 3 bottles of juice.

c. Describe all of the solutions for the equation and the corresponding graph of the points that are solutions.

Answer: The solution will be any whole number of glasses of water—or even fractions, but always 3 bottles of juice. The graph will be a horizontal line going through all of the points where the y-value is 3.

d. Create a context for an equation where the points in the solution for the equation would graph as a vertical line.

2. Decide whether the following are sometimes true, always true, or never true. Be ready to explain your reasoning.

a. The expression $4x + 3y$ will have the same value as long as an increase of 3 in the value of x is offset by a decrease of 4 in the value of y.

Answer: Always true, because when the exchange between 4 and 3 is maintained, the total value will be constant.

b. If you change $4x + 3y = c$ to $4x + 3y = d$ where d is different than c, the line graphs of the solution sets will be parallel.

Answer: Always true, because in both cases an increase of 3 in the value of x is offset by a decrease of 4 in the value of y and going the other way, a decrease of 3 the value of x is offset by a increase of 4 in the value of y.

c. If a is not zero and you solve an equation of the form $ax = b$ or $x + a = b$ for x, you will have exactly one solution.

Answer: True, because only one point will make the equation true; other points will either make the equation more than or less than b.

d. If a and b are not zero, then an equation of form $ax + by = c$ will have exactly one solution for x and y.

Answer: Never true, because the solution will be a set of points on a line; for example, (6, 3) will be one a point on the line that makes the equation true but so will the point (9, 1).

e. The solution to an equation of the form $2x + 5y = c$ is a point on a line.

Answer: Never true, because the solution will be the entire set of points on a line.