Building Concepts: Visualizing Linear Expressions

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students will use the distributive property to rewrite expressions for a given context in order to make connections between the scenario and the expression(s) that describe it.

The distributive property relates the addition and multiplication of real numbers and is central in creating equivalent expressions involving those operations.

Learning Goals

1. Find equivalent expressions involving the distributive property;
2. relate an area model to the distributive property;
3. recognize that the statement of the distributive property is symmetric;
4. interpret visual representations of the distributive property, i.e., $ax + bx = (a+b)x$ and $a(b+c) = ab + ac$.

Prerequisite Knowledge

Visualizing Linear Expressions is the ninth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students use the distributive property to rewrite expressions. Prior to working on this lesson, students should have completed *Using Structure to Solve Equations and Linear Inequalities in One Variable*. Students should understand:

- how to apply the distributive law and
- how to solve and justify the solution for a linear inequality in one variable.

Vocabulary

- **linear expressions:** two expressions set equal to each other and containing one or two variables and no variable is raised to an exponent greater than one, or used in the denominator of a fraction
- **distributive property:** used to multiply a single term and two or more terms inside parentheses

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Visualizing Linear Expressions_Student.pdf
- Visualizing Linear Expressions_Student.doc
- Visualizing Linear Expressions.tns
- Visualizing Linear Expressions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

Understanding variables as numbers helps students extend their work with the properties of operations from arithmetic to algebra. The distributive law is of fundamental importance in doing mathematics because it is the only property that connects the operation of multiplication to the operation of addition. For example, students who are accustomed to mentally calculating $5(37)$ as $5(30)+5(7)=150+35$ can see that $5(3a+7)=15a+35$ for all numbers *a*. Collecting like terms, e.g., $5b+2b-5p+3p-6b$ to make $1b-2p$, should be seen as an application of the distributive law. When students begin looking at products of two binomial terms such as $(x+2)(x-8)$, they apply the distributive law to get $x(x-8)+2(x-8)=x^2-8x+2x-16$ or $(x+2)(x^2-8x+4)$ to get $x(x^2-8x+4)+2(x^2-8x+4)$ or $x^2-8x^2+4x+2x^2-16x+8$. Students should recognize that the statement of the distributive property is symmetric (that is, $a(b+c)=ab+ac$ or $ab+ac=a(b+c)$) and be able to reason that $6x+8x^2$ can be written as $2x(3+4x)$.

Using mnemonics such as FOIL to develop the multiplication of binomials can lead to confusion and misunderstandings. FOIL can be difficult to interpret and does not extend to finding the product of a binomial and a trinomial or of two trinomials, both of which can be done by repeated applications of the distributive property. A better approach is to use “each times every”, which describes the distributive process and generalizes.

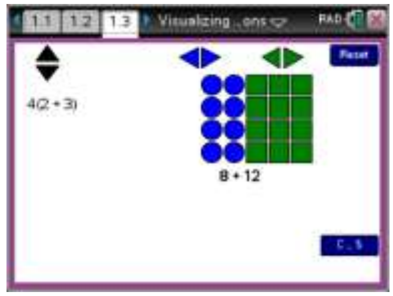
When students are comfortable with rational numbers including operations with negatives, they start to simplify general linear expressions that are a sum of terms involving rational numbers and variables such as *x* or *y*. Linear expressions have operations whose transformation may require an understanding of the rules for multiplying negative numbers, such as $7-2(3-9x)$. Typical mistakes when rewriting such an expression include ignoring the order of operations or not handling the negative appropriately. This activity emphasizes that different ways of writing expressions provide different ways of seeing a problem and that rewriting an expression in different forms in a problem context can make the connection between the expressions and the context visible.

Part 1, Page 1.3

Focus: The distributive property $a(b+c)=ab+ac$ where a is a positive whole number can be understood as repeated multiples of a sum.

On page 1.3, the up/down or black arrows change the number of rows; the right/left arrows or the blue/green arrows change the numbers of circles and squares in each row. The button **c, s** displays general expressions. **Values** displays numbers for *c* and *s*. Students can use the up/down arrows and keypad or **menu> Change Values** to change the values of c and s.

Reset returns to the original screen.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the arrows and buttons.

enter activates a button.

ctrl del resets the page.

Class Discussion

Have students… | **Look for/Listen for…**

Use the black arrows on the top left.

- ***Explain what the arithmetic expression $4(2+3)$ represents.***

  Answer: The expression indicates there are 4 rows each with 2 circles and 3 squares.

- ***What do the numbers 8 and 12 represent under the circles and squares?***

  Answer: A total of 8 circles and 12 squares in the 4 rows of 2 circles and 3 squares.

- ***Predict how many squares and circles you will have if you change the expression to generate 6 rows. Check your prediction using the TNS activity.***

  Answer: You will have 12 circles and 18 squares for 6 rows and 2 circles and 3 squares for 1 row.

- ***Find the number of rows of the expression if you have a total of 6 circles and 9 squares.***

  Answer: 3 rows or $3(2+3)$ will give you 6 circles and 9 squares.

Change the number of rows to 1, the number of circles to 4, and the number of squares to 5.

- ***Predict the number of circles and number of squares for 5 rows of $(4+5)$. Check your answer using the TNS activity.***

  Answer: 20 circles and 25 squares

Class Discussion (continued)

• ***Predict the number of rows you will have for a total of 16 circles and 20 squares. Check your answer using the TNS activity.***	Answer: 4 rows
• ***How does the total number of circles and squares change as the number of rows increases and decreases by 1? Explain why.***	Answer: The total number of circles will change by 4 and the total number of squares by 5 because each row has 4 circles and 5 squares.
• ***How does $3(4+2)$ relate to 12 circles and 6 squares?***	Answer: The product of 3 and 4 represents 12 circles and the product of 3 and 2, 6 squares.
The distributive property of multiplication over addition can be written as $a(b+c)=ab+ac$ or as $ab+ac=a(b+c)$. Use the TNS activity to help you answer each of the following:	
• ***Write an equation relating 4 rows of 2 circles and 3 squares to the total number of circles and total number of squares.***	Answer: The total number of circles would be 8 and the total number of squares would be 12, so $4(2+3)=4(2)+4(3)=8+12$.
• ***Use the distributive property to relate 10 circles and 4 squares to some number of rows made up of circles and squares.***	Answer: 2 rows of 5 circles and 2 squares: $10+4=2(5)+2(2)=2(5+2)$
• ***Write a statement using the distributive property that would produce a total of 12 shapes (circles and squares) if you know you have exactly 2 circles in a row.***	Answers will vary: 4 rows of 2 circles and 1 square: $4(2+1)=4(2)+4(1)$; 3 rows of 2 circles and 2 squares for $3(2+2)=3(2)+3(2)$; 2 rows of 2 circles and 4 squares for $2(2+4)=2(2)+2(4)$.

Student Activity Questions—Activity 1

1. Reset page 1.3. Select the button *c, s*. Each equation below is a statement of the distributive property. Use the TNS activity to find the values for each of the blanks.

a. $3(\_\_+\_\_)=18c+6s$

Answer: $3(\underline{6c}+\underline{2s})=18c+12s$

b. $8c+\_\_s=4(\_\_+3s)$

Answer: $8c+\underline{12}s=4(\underline{2c}+3s)$

c. $\_\_(4c+\_\_s)=20c+25s$

Answer: $\underline{5}(4c+\underline{5}s)=20c+25s$

Student Activity Questions—Activity 1 (continued)

2. **Reset page 1.3 and select *values*.**

   a. **Describe the difference in the ways used on the left and on the right to find the total value for each expression.**

   Answer: On the left, the values of the expressions inside the parentheses are computed first by substituting in the values of *c* and *s*. On the right, the parentheses are removed first using the distributive property and then the values of *c* and *s* are substituted.

   b. **Keep the same values for the circle and square. Explain how to change the number of circles or squares to get a total of 104.**

   Answers may vary. 4 rows with 4 circles per row and three squares per row.

   c. **Reset the page. Predict what will change if you increase the values in the squares to 3. Check your prediction by making $s = 3$.**

   Answers may vary. Only the values that involve *s* will change; the left side will be $4(2 \cdot 5 + 3 \cdot 3) = 4 \cdot 19 = 76$, and the right side will be $8 \cdot 5 + 12 \cdot 3 = 40 + 36 = 76$, in each case a total value of 76.

   d. **Predict what will change if the value of each circle is 9, and the value of each square remains 3.**

   Answer: The left expression will be $4(18 + 9)$ and the right expression will be $8(9) + 12(3)$ for a total value of 108.

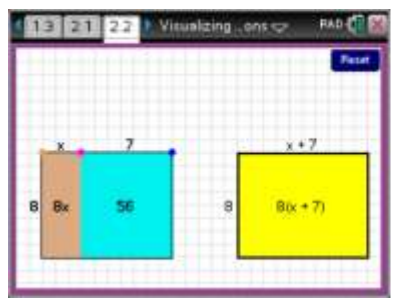
Part 2, Pages 2.2 and 2.3

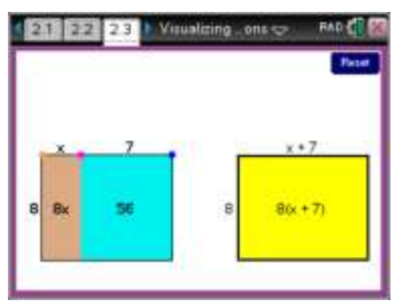
Focus: The distributive property can be represented by an area model.

On page 2.2, students can **tab** to or grab and drag the orange dot or the up/down arrows to change the height of the rectangles; grabbing and dragging the pink dot or using the right/left arrows changes the value of *x*; grabbing and dragging the blue dot or using the right /left arrows changes the width of the aqua rectangle.

Reset returns to the original screen.

Page 2.3 functions in the same way as page 2.2.





Class Discussion

Have students…	Look for/Listen for…
Consider the two rectangles on page 2.2. Note that you can tell the value of* x *from the scale.	
• ***How many small squares are in the aqua and brown rectangles on the left?***	Answer: The brown rectangle has 24 squares, and the aqua rectangle has 56 squares.
• ***Explain how you would find the area of each of the three rectangles (brown, aqua, and yellow). How is the area related to the number of small squares in the question above?***	Answer: The area of the brown rectangle is $8x = 8(3) = 24$ square units; the aqua rectangle has area 56 square units because it is $8(7)$; the area of the yellow rectangle is $8(3+7) = 8(10) = 80$ square units. The number of unit squares in the rectangle is the area.
• ***How are the areas of the brown and aqua rectangles related to the area of the yellow rectangle? Explain your reasoning.***	Answer: The sum of the areas of the brown and aqua rectangles is $24 + 56 = 80$ square units; the area of the yellow rectangle is also 80 square units because $8x + 56 = 80$ when $x = 3$.
• ***Move the orange dot to 12. Which set of rectangles $(12x + 74)$ or $12(x+7)$ will be easier to use to answer the question, how much did the area increase when the height became 12?***	Answers may vary: $12(3+7) = 12(10) = 120$ is easier to use because you can do the arithmetic in your head to get the areas of the rectangles are 40 square units more for a height of 12 than for a height of 8.
Reset the page. For each of the following, create the area by moving the orange dot to change the height, the pink dot to change the value of* x *or the blue dot to change the constant. Then, write an equation relating the three areas.	
• ***The area of the brown rectangle is 12 square units, and the area of the aqua rectangle is 42 square units.***	Answer: Changing the height to 6 and the value of *x* to 2 will make the brown rectangle 12 square units with an equation $6(2) + 6(7) = 6(2+7) = 54$.
• ***The area of the yellow rectangle is 90 square units, and the area of the aqua rectangle is 50 square units.***	Answer: Change the height to 10, the constant to 5 and the *x* to 4. The equation will be $10(4+5) = 10(4) + 10(5) = 90$.
• ***The area of the brown rectangle is 18 square units, and the area of the yellow rectangle is 90 square units.***	Answer: Change the height to 9, the *x* to 2 and the constant to 8. The equation will be $9(2) + 9(8) = 18 + 72 = 9(2+8) = 90$.

Class Discussion (continued)

Move to Page 2.3.

For each of the following, find the areas of the other rectangles and write an equation relating the areas of the three rectangles. (Note that the value of x is not visible from a scale on page 2.3 so answers will be in terms of x.)

- ***The area of the yellow rectangle is $6(x+8)$ square units.***

  Answer: The area of the brown rectangle is $6x$ square units and of the aqua rectangle 48 square units. The equation: $6x+48=6(x+8)$.

- ***The area of the aqua rectangle is 24 square units.***

  Answers may vary: The brown rectangle could be 8x square units and the area of the yellow rectangle $8(x+3)$. The equation: $8x+24=8(x+3)$.

- ***The area of the brown rectangle is $\left(\frac{11}{2}\right)x$ square units, and the area of the aqua rectangle is 33 square units.***

  Answer: The area of the yellow rectangle is $\frac{11}{2}(x+6)$; the equation is $\frac{11}{2}x+33=\frac{11}{2}(x+6)$.

- ***The area of the aqua rectangle is $\frac{165}{4}$ square units.***

  Answers may vary. The area of the brown rectangle could be 15/2 square units and the area of the yellow rectangle $\frac{15}{2}\left(x+\frac{11}{2}\right)$. The equation: $\frac{11}{2}x+\frac{165}{4}=\frac{11}{2}\left(x+\frac{15}{2}\right)$.

Student Activity Questions—Activity 2

1. Make the height of the brown rectangle 4.

a. What will the area of the yellow rectangle be if the area of the aqua rectangle is 22?

Answer: The area will be $4\left(x+\frac{11}{2}\right)$.

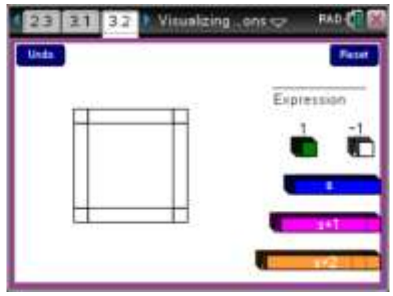
b. Move the pink dot to the left. Describe the change in the areas of the rectangles and in the expressions for the areas for each rectangle. Explain why your answer makes sense.

Answer: The expressions do not change; but as x decreases, the areas of the brown rectangle and the yellow rectangle get smaller because x is the width of the brown rectangle and part of the width of the yellow rectangle. The heights of the aqua and yellow rectangle remain the same as does the width of the aqua rectangle because the value of x is all that is changing. The area of the aqua rectangle remains the same because its dimensions, $\frac{11}{2}$ and 4, are not changed.

Part 3, Page 3.2

Focus: The distributive property can be used to write equivalent expressions involving the operations of multiplication and addition.

On page 3.2, selecting and dragging moves the boards. **Undo** undoes the recent moves in reverse order.



TI-Nspire Technology Tips

tab cycles through the board types.

enter selects a highlighted board or puts it in place.

Arrow keys move the selected board.

ctrl del resets the page.

Teacher Tip: The focus of pages 3.2 and 3.3 is on creating equivalent expressions involving the distributive property. Students find boards of appropriate lengths to make a border around a square with sides of length *s*, and examine the corresponding algebraic expressions. The distributive property is central to thinking about which expressions will be equivalent to those that can be used to build the border. On page 3.2, they choose boards of a given length and drag them to the border, considering both different configurations that will complete the border and expressions that are equivalent to $4s+4$. Here students have an unlimited supply of boards in stacks and use them until the border is filled in and an expression is generated according to the boards used. On page 3.3, students have to figure out what they need in advance by thinking about the expression and mentally picturing the border as they create expressions they think will satisfy the conditions necessary to build the border. Note that the stack of boards on page 3.3 gives them a partial mental image to help them move to the abstract representations.

Class Discussion

A square pool has a side length* s*. The border around the square can be made by using boards of different lengths, all with a width of 1 unit. The shortest board has a length of 1 unit.

- ***Select a board of length* s *and move it to the border. Select another board of length* s*, then two boards of length* s+1*. Describe the expression on the blank at the upper right.***

Answer: The expression is $s+s+(s+1)+(s+1)$.

Class Discussion (continued)

• ***What do you need to add in order to complete the border? What is the expression for the completed border?***	Answer: You need two small unit squares. The expression for the whole border will be $s+s+(s+1)+(s+1)+1+1$.
• ***Reset the page. Use two boards of length $s+2$. How can you complete the border? What is the final expression for the border?***	Answer may vary. Using two lengths of s will complete the border with $(s+2)+(s+2)+s+s$.
• ***How are the expressions in the two previous questions related?***	Answer: The expressions are equivalent. If you rearrange them and collect like terms, both will be $4s+4$.

Student Activity Questions—Activity 3

1. **Reset page 3.2.**

   a. **Using only boards of length s or smaller, write an expression that would describe how you could make the border. Drag the boards to see if your expression works.**

   Answer: The expression would be $4s+4$.

   b. **What is the fewest number of boards you can use to make the border? Write the expression.**

   Answers will vary: four boards of length $s+1$ for the expression $(s+1)+(s+1)+(s+1)+(s+1)$ or two boards of length $s+2$ and two boards of length s for the expression $(s+2)+(s+2)+s+s$.

   c. **Jerine wanted to make the border using a collection of boards only one of which has length $s+2$. Could she make the border? Why or why not?**

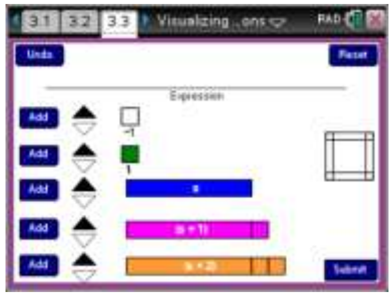
   Answer: She could make the border in several ways: for example, $(s+2)+s+s+s+2$.

   d. **Trey claimed the expressions used in the answers for the questions above are all equivalent. Do you agree with Trey? Why or why not?**

   Answer: Trey is correct because using the distributive property of multiplication over addition along with the other properties of addition and multiplication, all of the expressions are equivalent to $4s+4$.

Part 3, Page 3.3

Focus: The distributive property can be used to write equivalent expressions involving the operations of multiplication and addition.



On page 3.3, students can use **tab** to choose a board length; up arrow to select a number of boards of that length.

Add builds the expression.

TI-Nspire Technology Tips

menu accesses page options.

ctrl del resets the page.

Submit or **menu> Submit** shows the figure, the boards, and the corresponding expression.

Edit allows the boards and expression to be changed. The boards move to the square as they did on page 3.2.

Class Discussion

The arrows allow you to choose a number of boards of a given length to make the border for the square on page 3.2. Add displays the corresponding expression in the blank at the top.

- ***Use the arrows to choose four boards of length*** **s** ***and Add; then choose four boards of length 1 and Add. What expression did you create? Do you think you can complete the border? Explain why or why not. Submit to check your thinking.***

  Answer: $4s+4$. This will complete the border because it consists of 4 lengths of *s*, one for each side and 4 unit squares for the corners.

- ***Timon did not use the arrows. He selected Add*** **s** ***four times, Add 1 four times. Explain whether he will be able to complete the border. How will his expression differ from the expression in the question above?***

  Answer: He will be able to complete the border because his expression will still be equivalent to $4s+4$. His expression will be $s+s+s+s+1+1+1+1$.

- ***Sanu selected 2 boards of length*** $s+2$ ***and 2 boards of length*** **s*****. What expression will describe the total lengths of the boards he selected? Will he be able to complete the border? If not, explain how he could use Edit and adjust his choices.***

  Answer: The expression will be $2(s+2)+2s$, which is equivalent to $4s+4$. He will be able to complete the border.

Class Discussion (continued)

• ***Troy selected 3 boards of length $s+1$ and 1 board of length $s+2$. What expression will describe the total length of the boards he selected? Will he be able to complete the border? If not, explain how he could use Edit and adjust his choices.***	Answers may vary. The expression will be $3(s+1)+(s+2)$), which will be too long for the border by 1 unit, because it is equivalent to $4s+5$. He could use a white piece, which would be like sawing off the end of the board.
Explain how you could build the border using each of the following. (You can use other lengths as well.) Use the TNS activity to check your answer.	
• $s+2$	Answers may vary: Use another $s+2$ and 2 is to have $(s+2)+(s+2)+2s=4s+4$.
• $3s+2$	Answers may vary: Add *s* + 2 to get $(3s+2)+(s+2)=4s+4$.
• $2(s+1)$	Answers may vary: Add 2*s*, 1, and 1 so the total expression would be $2(s+1)+2s+1+1=4s+4$.
• $2s+2(s+1)$	Answer: Add 1 twice so the total expression would be $2s+2(s+1)+1+1=4s+4$.

Student Activity Questions—Activity 4

1. Think about the distributive property and decide which of the lengths represented by the given expressions can be used to make the border. If a combination does not make the border, explain how to add or subtract the fewest number of pieces to make it work.

a. $3(s+1)+1$

Answer: The expression is equivalent to $3s+4$, so another s is needed to make the border: $3(s+1)+1+s=4s+4$.

b. $2s+2s+4$

Answer: The expression is equivalent to $4s+4$ and will complete the border.

c. $1+2(s+1)$

Answer: The expression is equivalent to $2s+3$, so you need another $s+1$ and an *s* to complete the border.

Student Activity Questions—Activity 4 (continued)

2. Think about building the border and decide whether you agree or disagree with each of the following. Explain why or why not.

a. Sophie claimed that $1+3(s+1)$ is equivalent to $4(s+1)$.

Answer: Sophie is incorrect because one board is of length 1 and the 3 indicates 3 boards of length $s+1$, which is not the same as 4 boards of length $s+1$.

b. Bern said that $2s+4$ would be the same as $2(s+2)$.

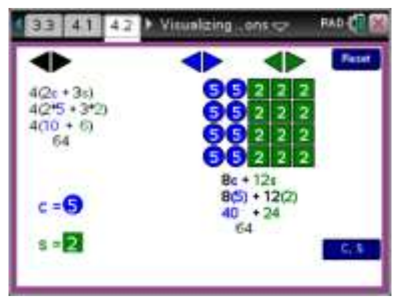
Answer: Two boards of length *s* and 4 boards of length 1 will be equivalent to 2 boards of length $(s+2)$ because $2(s+2)=2s+4$ by the distributive property.

c. Singe said that $2(s+1)+2(s+1)=4s+1$.

Answer: Singe is correct, because by the distributive property, $(2+2)(s+1)=4(s+1)$.

Part 4, Page 4.2

On page 4.2, the arrows function in the same way as those on page 1.3. The values for the circles and squares can be positive or negative. To change the values, select the circle or square and enter a new value or use **menu> Change Values**.



Teacher Tip: Part 4 is designed for students who can compute with integers. The question in Part 4 uses page 4.2 and allows students to replace the number of dots in the circles and squares with positive and negative numbers. The questions provide practice computing with integers and using the distributive property. You may want to have students create their own questions and exchange them with a partner.

Class Discussion

Have students…	Look for/Listen for…
Select Values. Let $c=2$ and $s=3$. Find numbers for the blanks to make a true statement. Use the TNS activity to guide your thinking.	
• $2(\_\_c+\_\_s)=\_\_+18=14$	Answers may vary. $2(1c+3s)=4+18=14$
• $\_\_(4c+\_\_s)=-32+\_\_=-8$	Answers may vary. $4(4c+2s)=-32+24=-8$

Class Discussion (continued)

- ***$6(\_\_c + \_\_s) = -36 + \_\_(3) = 36$***

Answers may vary.
$6(3c+4s) = -36 + 24(3) = 36$

- ***$\_\_c + 5s = 5(\_\_c + \_\_s) = -5$***

Answers may vary. $10c + 5s = 5(2c + 1s) = -5$

Deeper Dive

Describe the difference in the expressions $4(s+1)$ and $(s+1)(4)$.

If you add two inequalities with the same inequality sign, will the solution sets be the union of the two solution sets? Explain why or why not.

Answers may vary. One way to think is $4(s+1)$ is 4 rows of $s+1$ while $(s+1)4$ is $s+1$ rows of 4, which will be the same just arranged differently. Another way is to think about $4(s+1)$ as a rectangle with base 4 and height $(s+1)$, while (s+1)(4) is a rectangle with base $s+1$ and height 4, both of which will produce the same area.

In how many different ways is it possible to build the border on either page 3.2 or 3.3 using only two colors, if having the same color on the top and on the bottom is not considered to be different than having the same color on the sides and no boards are too long? Explain your reasoning.

Answer: Using 4 boards of length s and 4 boards of length 1 will create a border in blue and green, $4s+4$; 2 boards of length $(s+2)$ and 2 boards of length s,

$2(s+2)+2s$, a border in blue and orange. Any other combination will take more than two colors.

Write out the distributive property in each case:

- ***The total value in 4 rows of 3 circles, 5 squares and 6 triangles if* c *is the value of a circle,* s *the value of a square and* t *the value of a triangle.***

Answer: $4(3c+5s+6t) = 12c + 20s + 24t$

- ***The length of all the boards in $(s+1)$ rows of 2 boards of length* s *and 3 boards of length 1.***

Answer:
$(s+1)(2s+3(1)) = (s+1)(2s) + (s+1)(3)$

- ***The area of a rectangle that has a height of $s+2$ and a base of $s+3$.***

Answer:
$$\begin{aligned}(s+2)(s+3) &= s(s+3) + 2(s+3)\\ &= s^2 + 2s + 3s + 6\\ &= s^2 + 5s + 6\end{aligned}$$

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Select all the expressions that are equivalent to $8(t+4)$.

   a. $2(4t+2)$

   b. $8t+32$

   c. $4t+4+4t$

   d. $(8+t)+(8+4)$

   e. $(8\times t)+(8\times 4)$

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   Answer: b, e

2. Select the expression that is equivalent to $48+12$.

   a. $6(8+6)$

   b. $12(4+1)$

   c. $4(44+3)$

   d. $8(6+4)$

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   Answer: b

3. Determine whether each expression is equivalent to $6+3(x+2)$.

   a. $9(x+2)$

   b. $3x+12$

   c. $3(x+4)$

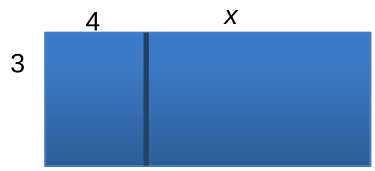
   d. $2(x+3)+6$

   Answer: b and c

4. Fill in the missing value so that the two expressions are equivalent: $3(s+5)+(s+5)$ and $\_\_(s+5)$.

   Answer: 4

5. Write two expressions to represent the area in the figure below.



Answer: $\boldsymbol{12+3x}$ ***and*** $\boldsymbol{3(4+x)}$

6. Enter the value of p so the expression $\frac{5}{6}-\frac{1}{3}n$ is equivalent to $p(5-2n)$.

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Answer: $\boldsymbol{p=\frac{1}{6}}$

7. Which expression is equivalent to $-8(10x-3)$?

a. $-80x+24$

b. $-80x-24$

c. $-80x-3$

d. $-80x+3$

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Answer: a

8. Indicate whether each expression in the table is equivalent to $\frac{1}{2}x-1$, equivalent to $x-\frac{1}{2}$, or not equivalent to $\frac{1}{2}x-1$ or $x-\frac{1}{2}$.

Expression	Equivalent to $\frac{1}{2}x-1$	Equivalent to $x-\frac{1}{2}$	Not equivalent to $\frac{1}{2}x-1$ or $x-\frac{1}{2}$
$\frac{2}{3}\left(\frac{3}{4}x-\frac{3}{2}\right)$			
$(2x+1)-\left(x+\frac{3}{2}\right)$			

PARCC VF888892 grade 7

Answer:

Expression	Equivalent to $\frac{1}{2}x-1$	Equivalent to $x-\frac{1}{2}$	Not equivalent to $\frac{1}{2}x-1$ or $x-\frac{1}{2}$
$\frac{2}{3}\left(\frac{3}{4}x-\frac{3}{2}\right)$	✓		
$(2x+1)-\left(x+\frac{3}{2}\right)$		✓	

9. Determine which expression is equivalent to $\frac{3}{4}-x\left(\frac{1}{2}-\frac{5}{8}\right)+\left(-\frac{3}{8}x\right)$.

   a. $-\frac{3}{4}x$

   b. $\frac{1}{2}x$

   c. $\frac{1}{8}-\frac{7}{8}x$

   d. $\frac{3}{4}-\frac{1}{4}x$

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Answer: d

Building Concepts: Visualizing Linear Expressions

TEACHER NOTES

Student Activity Solutions

In these activities, you will use the distributive property to rewrite expressions for a given context. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Reset page 1.3. Select the button **c, s**. Each equation below is a statement of the distributive property. Use the TNS activity to find the values for each of the blanks.

   a. $3(\_\_+\_\_)=18c+6s$

   Answer: $3(\underline{6c}+\underline{2s})=18c+12s$

   b. $8c+\_\_s=4(\_\_+3s)$

   Answer: $8c+\underline{12}s=4(\underline{2c}+3s)$

   c. $\_\_(4c+\_\_s)=20c+25s$

   Answer: $\underline{5}(4c+\underline{5}s)=20c+25s$

2. Reset page 1.3 and select **values**.

   a. Describe the difference in the ways used on the left and on the right to find the total value for each expression.

   Answer: On the left, the values of the expressions inside the parentheses are computed first by substituting in the values of c *and* s*. On the right, the parentheses are removed first using the distributive property and then the values of* c *and* s *are substituted.*

   b. Keep the same values for the circle and square. Explain how to change the number of circles or squares to get a total of 104.

   Answers may vary. 4 rows with 4 circles per row and three squares per row.

   c. Reset the page. Predict what will change if you increase the values in the squares to 3. Check your prediction by making $s=3$.

   Answers may vary. Only the values that involve s will change; the left side will be $4(2\cdot5+3\cdot3)=4\cdot19=76$*, and the right side will be* $8\cdot5+12\cdot3=40+36=76$*, in each case a total value of 76.*

   d. Predict what will change if the value of each circle is 9, and the value of each square remains 3.

   Answer: The left expression will be $4(18+9)$ *and the right expression will be* $8(9)+12(3)$ *8(9) +12(3) for a total value of 108.*

Activity 2 [Page 2.3]

1. Make the height of the brown rectangle 4.

   a. What will the area of the yellow rectangle be if the area of the aqua rectangle is 22?

   Answer: The area will be $4\left(x+\frac{11}{2}\right)$.

   b. Move the pink dot to the left. Describe the change in the areas of the rectangles and in the expressions for the areas for each rectangle. Explain why your answer makes sense.

   Answer: The expressions do not change; but as x *decreases, the areas of the brown rectangle and the yellow rectangle get smaller because* x *is the width of the brown rectangle and part of the width of the yellow rectangle. The heights of the aqua and yellow rectangle remain the same as does the width of the aqua rectangle because the value of* x *is all that is changing. The area of the aqua rectangle remains the same because its dimensions,* $\frac{11}{2}$ *and 4, are not changed.*

Activity 3 [Page 3.2]

1. Reset page 3.2.

   a. Using only boards of length *s* or smaller, write an expression that would describe how you could make the border. Drag the boards to see if your expression works.

   Answer: The expression would be $4s+4$.

   b. What is the fewest number of boards you can use to make the border? Write the expression.

   Answers will vary: four boards of length $s+1$ *for the expression* $(s+1)+(s+1)+(s+1)+(s+1)$ *or two boards of length* $s+2$ *and two boards of length s for the expression* $(s+2)+(s+2)+s+s$.

   c. Jerine wanted to make the border using a collection of boards only one of which has length $s+2$. Could she make the border? Why or why not?

   Answer: She could make the border in several ways: for example, $(s+2)+s+s+s+2$.

   d. Trey claimed the expressions used in the answers for the questions above are all equivalent. Do you agree with Trey? Why or why not?

   Answer: Trey is correct because using the distributive property of multiplication over addition along with the other properties of addition and multiplication, all of the expressions are equivalent to $4s+4$.

Activity 4 [Page 3.3]

1. Think about the distributive property and decide which of the lengths represented by the given expressions can be used to make the border. If a combination does not make the border, explain how to add or subtract the fewest number of pieces to make it work.

   a. $3(s+1)+1$

   Answer: The expression is equivalent to $3s+4$, so another s is needed to make the border: $3(s+1)+1+s=4s+4$.

   b. $2s+2s+4$

   Answer: The expression is equivalent to $4s+4$ and will complete the border.

   c. $1+2(s+1)$

   Answer: The expression is equivalent to $2s+3$ so you need another $s+1$ and an s to complete the border.

2. Think about building the border and decide whether you agree or disagree with each of the following. Explain why or why not.

   a. Sophie claimed that $1+3(s+1)$ is equivalent to $4(s+1)$.

   Answer: Sophie is incorrect because one board is of length 1 and the 3 indicates 3 boards of length $s+1$, which is not the same as 4 boards of length $s+1$.

   b. Bern said that $2s+4$ would be the same as $2(s+2)$.

   Answer: Two boards of length s and 4 boards of length 1 will be equivalent to 2 boards of length $(s+2)$ because $2(s+2)=2s+4$ by the distributive property.

   c. Singe said that $2(s+1)+2(s+1)=4s+1$.

   Answer: Singe is correct because by the distributive property $(2+2)(s+1)=4(s+1)$.