Building Concepts: What Is a Solution to a System of Equations?

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students are introduced to a system of linear equations in two variables and find solutions using balances. Then students investigate graphically systems with a unique solution, no solutions, or infinitely many solutions.

Some systems of linear equations in two variables have one point as a solution, some have no points as a solution, and some have infinitely many points as a solution.

Learning Goals

1. Identify whether the solution to a system of two linear equations is a point, no solution, or all of the points on a line;
2. describe the solution to a system of two linear equations in two variables in terms of the graphs of the two equations;
3. identify characteristics of equations that will lead to graphs that are parallel, the same line or that intersect in one point;
4. recognize that two lines with the same slope are parallel if they have no point in common.

Prerequisite Knowledge

What Is a Solution to a System of Equations? is the sixteenth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students associate algebraic representations for two linear equations in two variables with balance scales, where they identify values for x and y that make the bars representing two equations balance. This lesson builds on the concepts of previous lessons. Prior to working on this lesson, students should have completed *Visualizing Equations Using Mobiles and Equations of the Form* $ax + by = c$.

Students should understand:

- how to identify a solution to an equation;
- how to represent a solution set graphically;
- that all of the points (x, y) that make $ax + by = c$ a true statement lie on a straight line.

Vocabulary

- **system of linear equations in two variables:** comprises two or more equations where a common solution to the equations is sought
- **intersection:** the point where two lines meet or cross
- **slope:** a number that describes both the direction and the steepness of the line
- **parallel lines:** two **lines** in a plane that do not intersect or touch each other at any point
- **solution to a system of two linear equations in two variables:** set of points that makes two equations true statements

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Lesson Pacing

This lesson may take 50–90 minutes to complete with students.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- What Is a Solution For a System of Equations_Student.pdf
- What Is a Solution For a System of Equations_Student.doc
- What Is a Solution For a System of Equations.tns
- What Is a Solution For a System of Equations_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

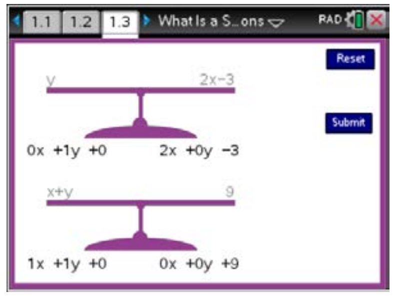
Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In earlier work, students identified a *solution* to an equation, if one exists, as a value that makes the equation true when substituted for the variable. In *Equations of the Form* $ax + by = c$*,* they identified the solution to a linear equation in two variables as the set of replacements for x and y that made the equation a true statement. They also represented this solution set graphically. In earlier informal work with systems of equations, *Visualizing Equations Using Mobiles*, students looked for the weights of shapes on mobiles that made the mobile balance. This lesson associates algebraic representations for two linear equations in two variables with balance scales, where students identify values for x and y that make the bars representing two equations balance. Building on the knowledge that all of the points (x, y) that make $ax + by = c$ a true statement lie on a straight line, the solution sets for each of the equations in a system of two linear equations can be represented as the graph of a straight line. The solution, or set of points that makes two equations true statements, if a solution exists, would be the point of intersection of the graphical representations of the two equations. If the graphs are parallel lines, the system has no solution (sometimes called inconsistent equations); if the two lines intersect in one point, that point will be the solution (independent systems), and if the lines coincide, the equations represent the same line and the solution is the set of all points on that line (dependent systems).

Part 1, Page 1.3

Focus: A solution for two equations in two variables *x* and *y* will be replacements for the variables that make both of the equations true at the same time.



On Page 1.3, students can select a number in an expression to edit the value.

Submit displays the balance bar for the resulting expressions.

Select and drag a point to move the value for *x* or *y*.

Edit returns to the screen where expressions can be changed.

Reset returns to the original screen and expressions.

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menu accesses page options.

tab moves among the values in the expressions and edit/submit button as well as switching among *x* and *y* dots.

ctrl del resets the page.

Class Discussion

Have students…	Look for/Listen for…
***On Page 1.3, select* Submit.**	
• ***Consider the balance bar on the left. What happens when you move x?***	Answers may vary. If you drag *x* to the right, the balance bar becomes more unbalanced; if you drag *x* to the left, the balance bar tips in the other direction.
• ***Move both* x *and* y *and see if you can make the balance bar on the left horizontal. Explain why some values make the balance bar on the left horizontal.***	Answers may vary. If $x=-1$ and $y=-5$, the expression on the left side of the balance bar is -5 and on the right side of the balance bar is $2(-1)-3=-2-3=-5$. Both sides of the expression have the same value, which is why the bar balances.
• ***Describe the balance bar on the right.***	Answers may vary. For $x=-1$ and $y=-5$, the bar is unbalanced with the value of the right side 9 and the left side -6, 15 units apart.

Class Discussion (continued)

• ***Drag*** x ***or*** y ***and find other values for*** x ***and*** y ***that make the balance bar on the left balance. What happened to the bar on the right?***	Answers will vary. $x = -2$ and $y = -7$ makes the bar balance with -7 on both sides. The bar on the right is more unbalanced with one side -9 and the other 9, 18 units apart.
Consider the bar on the right.	
• ***Find values for*** x ***and*** y ***that will make the bar on the right balance.***	Answers will vary. $x = 1$ and $y = 8$ will make the bar balance because the values on both sides will be 9.
• ***Find another set of values for*** x ***and*** y ***that will make the bar on the right balance.***	Answers will vary. Another set might be $x = 3$ and $y = 6$, which will make both sides equal 9.
• ***Find a set of values for*** x ***and*** y ***that will make both of the bars balance.***	Answer: When $x = 4$ and $y = 5$, both bars will balance.
• ***Find another set of values that will make both bars balance.***	Answer: No other values will make both bars balance. If you increase *y* by 1, to keep the right bar balanced, you have to decrease x by 1 (to keep the sum of 9), but that makes the left bar unbalanced. If you increase *x* by 1, to keep left bar balanced, you have to increase the *y* by 2, but that unbalances the right bar.
Select **Edit*****. Change the expressions on the first bar to*** $2x + y$ ***and*** $y + 10$ ***and leave the second bar as it was in the problem above.*** **Submit*****.***	
• ***Balance one of the bars. Explain which bar you choose to balance and why.***	Answers may vary. The bar on the left will balance for any *y*-value as long as $x = 5$, because the *y*-value will always add the same amount to both expressions and so could be subtracted from both expressions, leaving 2*x* on one side and 10 on the other.
• ***Find values for*** x ***and*** y ***that make both bars balance. Explain how you found the values.***	Answers may vary. If $x = 5$ on the left, then the right side of the bar on the left will be $5 + -1 = 4$ but the other end will be 9. Increasing *y* to 4 will make the bar on the left balance with both sides having values of 9.

Class Discussion (continued)

• ***Are these the only values for* x *and* y *that will balance both bars? How do you know?***	Answer. If x is any value other than 5, the left bar will not balance, regardless of the value for y. This makes the only possible value for y on the right bar to keep it balanced is $y = 4$. Only one possible pair of values makes both bars balance (5, 4).
Reset. Change the expressions on the first bar to* y *and to* $2x - 4$. *Change the expressions on the second bar to* $-x + 3y$ *and to* -7. *Submit.	
• ***What equations could you write to represent the two balance bars?***	Answer: $y = 2x - 4$ and $-x + 3y = -7$
• ***Find values to make the bar on the left balance. What does that suggest about the equation representing that bar?***	Answers will vary. One example is $y = 2x - 4$ $x = 0$ and $y = -4$. Those values make the equation true.
• ***If you increase* x *by one, how does* y *have to be changed so the bar remains balanced?***	Answer: If you increase x by 1, because y is twice the x-value, you have to increase y by 2.
• ***Explain how you could use the idea above as a strategy to find the values that make both bars balance.***	Answer: Systematically increase x by 1 and increase y by 2 to keep the left bar balanced until you find the values for x and y for which both of the bars balance. (Or increase x by 1 and decrease y by 2). The values that make both bars balance will be $x = 1$ and $y = 2$.
Select* Edit*. Change the second bar to represent the equation* $2x - y = -4$ *and leave the expressions on the top bar as in problem 4. Submit.	
• ***Find values for* x *and* y *that make the left bar balance.***	Answers will vary. The left bar will balance for $x = 0$ and $y = -4$.
• ***As* x *increases in the left bar, how does* y *have to change for the bar to stay balanced? Describe how the left bar changes.***	Answer: As x increases by 1, y has to increase by 2 to stay balanced. The right bar does not change, and the values of the two expressions stay at -4 and 4 or 8 units apart.
• ***Continue to change* x *by 1 unit and adjust* y *to keep the left bar balanced. How does the right bar change?***	Answer: The right bar is always unbalanced with the value of the expression at one end at -4 and the other at 4 or 8 apart.

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Class Discussion (continued)

- ***Find values that balance the right bar and observe what happens to the left bar for different values of* x *and* y *that make the right bar balance. What conclusion can you make about a pair of values that will make both bars balance?***

Answers may vary. The right bar will balance for $x = 0$ and $y = 4$. The values of the expressions on the right bar are 8 apart, from 4 to -4. The right bar balances for $x = -3$ and $y = -2$, but the values of the expressions on the left bar are 8 apart, at -10 and -2. The right bar balances for $x = 0$ and $y = 4$. No matter where the right bar balances, the values for each expression on the left bar are 8 apart. The two bars will never balance at the same time.

Select* Edit*. Make the second bar represent $2x - y = 4$, and leave the equation on the first as it was in the previous problem. Submit.

- ***Find values that make the right and left bar balance, that make both equations true.***

Answers may vary. The bars both balance at $x = 0$, $y = -2$.

- ***Find another pair of values that make the two bars both balance.***

Answers. They both balance at $x = 10$ and $y = 1$.

- ***What conclusion can you draw about the values that make both of the bars balance?***

Answers: Whenever one of the bars balances, the other bar will as well.

Student Activity Questions—Activity 1

1. Two equations involving the same variables are called a system of equations.

a. Recall, what is a solution to a linear equation of the form $ax + b = c$?

Answer: The solution is the value of *x*, if one exists, that makes the equation a true statement.

b. What do you think is meant by a solution to a system of two equations?

Answer: The solution to a system of equations will be the replacements for the variables that make both of the equations true at the same time.

c. *Reset* and *Submit*. Refer to the class discussion. What is the solution to the system $y = 2x = 3$ and $x + y = 9$? Explain why.

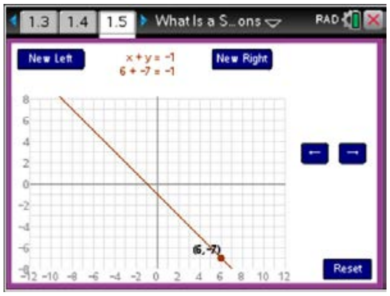
Answer: The solution would be the values for $x = 4$ and $y = 5$ because both of those values make both of the bars balance and they make the equations a true statement

d. Do you think every system of equations will have a solution? Why or why not?

Answers may vary. In the question 5 above, no values for *x* and *y* made the both bars balance at the same time, so no values for *x* and *y* made both of the equations true at the same time, which suggests there is no solution set.

Part 2, Page 1.5

Focus: The solution to a system of linear equations in two variables is a point, no points, or infinitely many solutions.



On Page 1.5, students can move the point by using the left and right arrow keys.

New Right adds an equation on the right, then cycles through five different equations and graphs.

New Left cycles through three equations and their graphs.

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menu accesses page options.

tab cycles among the new equation buttons, the arrows, and reset.

Note: on the vertical line, the up and down arrows on the keypad move the point along the line.

Reset clears the right equation and restores the original equation on the right.

Class Discussion

Have students…	Look for/Listen for…
Move the point on the line.	
• ***What do you notice about the values for* x *and* y *as the point moves?***	Answers may vary. Replacing the values for the point into the equation $x+y=-1$ make a true statement.
• ***Remember what a solution to a linear equation in two variables is. What will a solution look like for the equation $x+y=-1$? How do you know when you have a solution?***	Answer: The solution will be two numbers, *x* and *y*, that can be written as an ordered pair (*x, y*) representing a point on the line determined by $x+y=-1$. When specific values for (*x, y*) are a solution, replacing *x* and *y* in the equation by the values will produce a true statement.
• ***How many points will be in the solution set for $x+y=-1$? Explain your reasoning.***	Answer: Infinitely many points will be in the solution set. Every combination of *x* and *y* that adds to -1, such as $(\frac{1}{2},-\frac{3}{2})$, will be on the line because when used in the equation, they make a true statement.

Class Discussion (continued)

- ***What is the slope of the line determined by the graph of $x + y = -1$? Explain how you found your answer.***

  Answer: The slope is -1. You can find two points on the line and calculate the change in y over the change in *x*; you can count the slope using the grid lines on the graph. You can write the equation in the form $y = -1x + 1$ and take the slope from the multiplier of *x.*

Select **New Right.**

- ***Describe the difference in the ordered pairs for the points on the two lines.***

  Answer: The *x*-coordinates are the same, -6, and *y*-coordinates are 15 units apart (from 8 on the blue line to -7 on the brown line).

- ***Use the arrow keys to move the points. Describe what happens.***

  Answer: The points move together as the value for *x* approaches 1. The closer the two points are to the point of intersection for the two lines, the closer together the *y*-values for each point become.

- ***Find a value of* x *that makes the same* y*-value for each equation.***

  Answer: When $x = 1$, the *y*-value of -2 makes both of the equations true.

- ***How are the lines connected to the balance bars from Page 1.3?***

  Answer: Each line could be like one of the balance bars; when the bar is balanced, the two expressions for the bar have the same value or are equal like an equation. When both of the bars are balanced, it is like having the point (*x, y*) as the solution to the system of equations, a point that makes both of the equations true.

Consider the two lines $y = 2x - 4$ and $x + y = -1$.

- ***What are the slopes of the two lines? Explain how you found each.***

  Answer: The brown line, $x + y = -1$, still has slope -1, and the blue line, $y = 2x - 4$, has slope 2. The rate of change for $y = 2x - 4$ as x increases by 1 is 2 as you can see from the graph or by using the equation; when x goes from 1 to 2, the y goes from -2 to 0, for example.

Class Discussion (continued)

- ***Use the slope to explain why there is no other point that is a solution for the system of equations.***

Answer: At (1, -2), the lines intersect, and as *x* gets larger than 1, the slope of the blue line goes over 1 and up 2 while the brown line with slope -1 goes over 1 and down 1. So the lines will never intersect again, because the two lines are diverging.

Select New Right again. You should have the equations $x+y=-1$ and $y=2$.

- ***Describe the new line.***

Answer: It is a horizontal line.

- ***Move the points on the lines. What can you say about all of the points on the horizontal line? How do these points relate to the equation of the line?***

Answer: All of the points have a *y*-coordinate of 2. The equation of the line is $y=2$.

- ***What is the solution to the system of the two equations? How do you know you have the solution?***

Answer: (-3, 2) is the solution which makes the equation, $y=2$ true, and for the second, $x+y=-1$, $-3+2=-1$ is a true statement. The values for *x* and *y* for the point

(-3, 2) make both equations true, so the point is in the solution set. No other points are in the solution set because as *x* gets closer to -3 moving from the left to the right, the difference between the *y*-values on the blue line $y=2$ and the *y*-values on the brown line $x+y=-1$ get smaller until the difference is 0 at $x=-3$ and then the absolute value of the difference increases.

- ***Select* New Right *again. What is the difference between this system of equations and the previous system? (Note the vertical arrows on the handheld will move the point on the vertical line.)***

Answer: The vertical line has a constant value of $x=1$ for every value of *y*. The other system had a constant value for *y*, which was a horizontal line. The solution for this system will be (1, -2).

Select* New Right*. You should have the system $x+y=-1$ and $-x+3y=1$.

- ***Predict the solution set just by looking at the graph. How do you know?***

Answer: (-1, 0) because you can see from the graph that it will be the intersection point of the two lines; the values $x=-1$ and $y=0$ make both of the equations true.

Class Discussion (continued)

• ***Use the right arrow to move the points. What happens to the difference between the*** y-***values of the points as*** x ***increases?***	Answer: The difference, the brown *y*-value minus the blue *y*-value, decreases until you hit the intersection point where the difference is 0, and then the difference begins to increase.
• ***Is the point you identified the only solution? Explain why or why not.***	Answers may vary. Some might suggest that the only way two straight lines can behave is to intersect at one point or are parallel and never intersect. These two lines intersect at one point, so they are not parallel. Some might use the argument in above where the difference between the *y*-values will be 0 at only one point, which is the only point where, for the same *x*-value, both lines have the same *y*-value.
• ***What are the slopes of the two lines? Explain how you found your answers.***	Answer: The slope of $x+y=-1$ is -1 from previous work. The slope of $-x+3y=-1$ is $\frac{1}{3}$, which can be found using two points on the line such as (1, 0) and (4, 1) or by finding the rate of change in the y-values as the *x*-value increases by 1.

Reset the page. Select **New Left** ***to display the line representing*** $2x-y=4$ ***. Use the right arrow to move the points.***

• ***What is the slope of the line? Explain how you know.***	Answer: The slope is 2, because you can see from the graph that as x increases by 1, *y* increases by 2. (Reasons may vary.)
• ***Select*** **New Right** ***to display the system*** $2x-y=4$ ***and*** $y=2x-4$ ***. Describe what happened then move the point and describe how the equations change.***	Answer: Nothing happened to the graph except its color changes to indicate that both equations represent the same line. The coordinates of each point on the line are replaced in both equations and make both equations true.
• ***How does the slope of the line representing*** $y=2x-4$ ***compare to the slope you found above for*** $2x-y=4$ ***?***	Answer: The slopes of both lines are 2.

Student Activity Questions—Activity 2

1. **Do not *Reset*. For each of the following, predict the point of intersection of the lines representing the equations and predict the solution for the system. Then use the TNS activity to check your answer. (Note you can display the second equations by cycling through the New Right equations.**

   a. $\mathbf{2x - y = 4}$ **and** $\mathbf{y = 2}$

   Answer: $x = 3$, $y = 2$ and the lines will intersect at (3, 2).

   b. $\mathbf{2x - y = 4}$ **and** $\mathbf{x = 1}$

   Answer: $x = 1$, $y = -2$ and the lines will intersect at (1, -2).

   c. $\mathbf{2x - y = 4}$ **and** $\mathbf{-x + 3y = 1}$

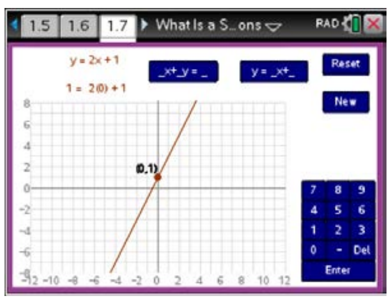
   Answer: You cannot tell for sure from the graph; the lines will intersect somewhere for *x* between 2 and 3 and a bit over 1 for *y*.

   d. $\mathbf{2x - y = 4}$ **and** $\mathbf{2x + 2y = 8}$

   Answer: $x = 0$, $y = -4$ and the lines will intersect at (0, -4).

Part 2, Page 1.7

Focus: The solution to a system of linear equations in two variables is a point, no points, or infinitely many solutions.



On Page 1.7, dragging or using the left/right arrow keys on the keypad will move the coordinate point. Select a form of a line to edit a second equation. Press **enter** to display the new line. Select and drag a point on the line to move.

New generates a new equation on the left.

Reset returns to the original equation.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles among equation values and then between the points.

Arrow keys move the points on the lines.

ctrl del resets the page.

Student Activity Questions—Activity 3

1. **The equation on the left is $y = 2x + 1$. Answer each of the following and explain how the graph can be used to check your answer.**

   a. **Select an equation on the right in the form $y =$ __ $x +$ __ and enter values in the blanks to create a system of equations that will not have a solution. Explain your reasoning.**

   Answers will vary. The multiplier of *x* as 2 will produce two lines with the same slope. $y = 2x + a$ for any value of *a* except 1 will be a line parallel to the given line and produce a system that will not have a solution.

   b. **Select *Edit* and change the equation on the right to one of the form __ $x +$ __ $y =$ __ so the system will have $x = -2$ as the *x*-coordinate of the point of intersection**.

   Answers will vary. Use $x = -2$ in the first equation to find the value of *y* that satisfies that equation, $y = -3$ and then choose a values for the blanks so that (-2, -3) make a true statement, for example, $5x - 2y = -4$. Both of the lines should contain the point (-2, -3).

   c. **Select *Edit* and change the equation on the right to one of the form __ $x +$ __ $y =$ __ so that two points that lie on the graph of $y = 2x + 1$ equation are in the solution of the system.**

   Answers may vary: $-2x + y = 1$ would work. There should only be one line in the graph.

2. **Reset Page 1.7. Select *New* until you generate a new equation on the left that passes through the vertical axis on the screen. Answer each of the following and explain how the graph can be used to check your answer. Hint: Grabbing and dragging the point on the equation might help you find an answer.**

   a. **Select an equation on the right in the form $y =$ __ $x +$ __ and enter values in the blanks to create a system of equations that will not have a solution. Explain your reasoning.**

   Answers will vary. The multiplier of *x* as 2 will produce two lines with the same slope. $y = 2x + a$ for any value of *a* except 1 will be a line parallel to the given line and produce a system that will not have a solution.

   b. **Select *Edit* and change the equation on the right to one of the form __ $x +$ __ $y =$ __ to produce a system of equations with the *x*-intercept of both equations as the solution. (If the point on the *x*-axis for the given line is not visible on the screen, generate a new line.)**

   Answers will vary. The point of intersection of the two lines should be on the *x*-axis with coordinates of the form (x, 0).

   c. **Select *Edit* and change the equation on the right to one of the form __ $x +$ __ $y =$ __ to produce a system of equations where one of the equations is a horizontal line.**

   Answers will vary. One of the equations has to be of the form $y =$ __ in order for the solution to lie on a horizontal line.

Student Activity Questions—Activity 3 (continued)

d. Select *Edit* and change the equation on the right to one of the form $y = \_\_ x + \_\_$ to produce an equation whose solution set is a point in the third quadrant.

Answers will vary. Drag the point on the line to a point in the third quadrant. For example, $4x + 5y = 17$ was the equation; drag the point to $(-1, -3)$ in the third quadrant and on the line. Create a new equation such that $(-1, -3)$ makes a true statement, for example $y = 4x + 1$.

3. Identify the following as true or false. Find an example from the TNS activity to support your thinking.

a. If a system does not have a solution, no value of *x* will make both equations in the system true.

b. If two lines intersect at the point (0, 0), the system will not have a solution.

c. If one equation is a multiple of the other, the system will have infinitely many solutions.

d. $x = 3$ and $y = 4$ is a system of equations with the solution (- -3, 4).

Answer: a) true; b) false; c) true; d) true

Deeper Dive

- ***Go to Page 1.5. Is the relationship defined by the line $x + y = 3$ a proportional relationship between* x *and* y*? Why or why not?***

  Answer: The relationship is not a proportional relationship between *x* and *y* because the points on the line are not associated with equivalent ratios; no multiplier exists that can multiply the ratio associated with the point (1, 2), namely 1:2, to get the ratio associated with the point (0, 3), 0:3. There is a proportional relationship between the change in *x* and the change in *y*, however.

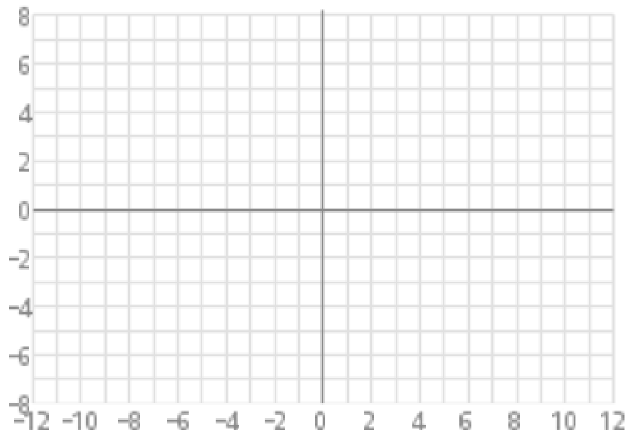
- ***How do you think the graphs of the following equations are related? $x + 2y = 8$; $2x - y = 12$, $3x + y = 20$ and $5x = 30$?***

  Answers may vary. All four of the equations intersect in the point (6, 2).

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Graph a system of two equations that has a single solution of $(-2, -3)$.



Grade 8 Mathematics Practice Test Scoring Guide,1866

***Answer:** Any two lines that intersect at $(-2, -3)$ including a horizontal line through $y = 3$ and a vertical line through $x = 2$.*

2. Joe solved this linear system correctly.

$6x + 3y = 6$

$y = 2x + 2$

These are the last two steps of his work.

$6x - 6x + 6 = 6$

$6 = 6$

Which statement about this linear system must be true?

a. x must equal 6

b. y must equal 6

c. There is no solution to this system.

d. There are infinitely many solutions to this system.

Smarter Balanced Grade 8 Mathematics Practice Test Scoring Guide, 1864

Answer: d

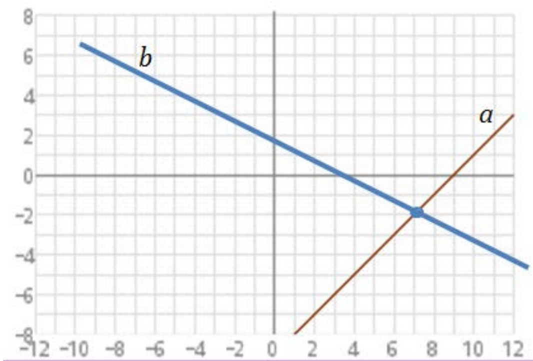
3. Line *a* is shown on the graph. Construct line *b* on the graph so that:

   Line *a* and line *b* represent a system of linear equations with a solution of $(7, -2)$.

   The slope of line *b* is greater than -1 and less than 0.

   The *y*-intercept of line *b* is positive.

Answer:



(line a is $y = x - 9$)

4. A system of equations is shown.

$x+\frac{1}{2}y=0$ and $x-\frac{3}{2}y=4$.

In the solution to this system of equations, what is the value of *y*?

PARCC Math Spring Operational 2015 Grade 8 End of Year Released Items

Answer: $\boldsymbol{y=2}$

5. Four systems of equations are shown in the table. Indicate which system has no solution, one solution or infinitely many solutions.

System of equations	$2x+3y=-6$ $4x+6y=-12$	$x=1$ $y=2$	$x-2y=4$ $x-2y=5$	$y=5x+20$ $3y=15x+60$
No solution				
One solution				
Infinitely many solutions				

PARCC Math Spring Operational 2015 Grade 8 End of Year Released Items

Answer:

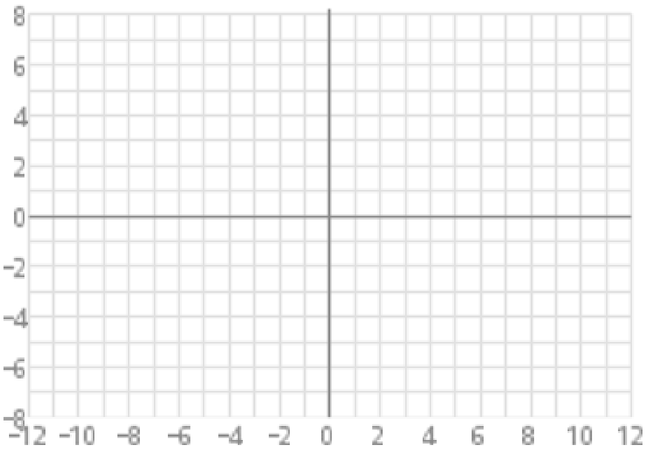
System of equations	$2x+3y=-6$ $4x+6y=-12$	$x=1$ $y=2$	$x-2y=4$ $x-2y=5$	$y=5x+20$ $3y=15x+60$
No solution			**X**	
One solution		**X**		
Infinitely many solutions	**X**			x

6. The equation of line *j* is $y = -2x + 8$

   The equation of line *k* is $y = 3x - 7$

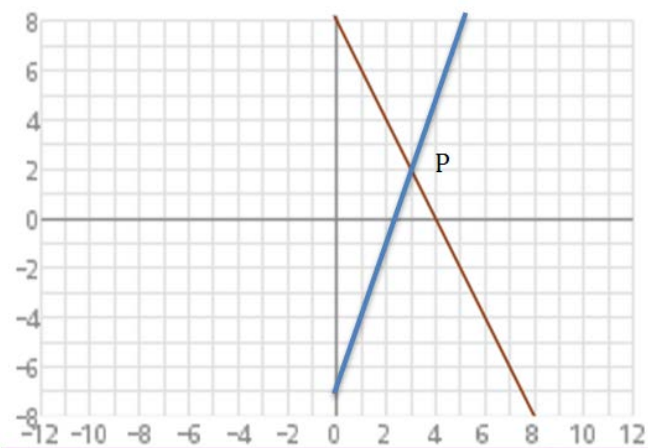
   The equations of lines *j* and *k* form a system of equations. The solution is located at Point P.

   Graph the system of equations on the coordinate plane. Plot Point P.



PARCC Math Spring Operational 2015 Grade 8 End of Year Released Items

Answer:



7. A system of two linear equations is graphed on a coordinate plane. If the system of equations has infinitely many solutions, which statement must be true?

   a. On the graph, there are no points (x, y) that satisfy both equations.

   b. On the graph there is exactly one point (x, y) that satisfies both the equations.

   c. On the graph, any point (x, y) that satisfies one of the equations cannot satisfy the other equation.

   d. On the graph, any point (x, y) that satisfies one of the equations must also satisfy the other equation.

   PARCC Math Spring Operational 2015 Grade 8 End of Year Released Items

   Answer: d

Building Concepts: What Is a Solution to a System of Equations?

TEACHER NOTES

Student Activity Solutions

In these activities, you will investigate systems of linear equations. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Two equations involving the same variables are called a system of equations.

   a. Recall, what is a solution to a linear equation of the form $ax+b=c$?

   Answer: The solution is the value of x, *if one exists, that makes the equation a true statement.*

   b. What do you think is meant by a solution to a system of two equations?

   Answer: The solution to a system of equations will be the replacements for the variables that make both of the equations true at the same time.

   c. **Reset** and **Submit**. Refer to the class discussion. What is the solution to the system $y=2x=3$ and $x+y=9$? Explain why.

   Answer: The solution would be the values for $x=4$ and $y=5$, because both of those values make both of the bars balance and they make the equations a true statement.

   d. Do you think every system of equations will have a solution? Why or why not?

   Answers may vary. In the question 5 above, no values for x *and* y *made the both bars balance at the same time, so no values for* x *and* y *made both of the equations true at the same time, which suggests there is no solution set.*

Activity 2 [Page 1.5]

1. Do not **Reset**. For each of the following, predict the point of intersection of the lines representing the equations and predict the solution for the system. Then use the TNS activity to check your answer. (Note you can display the second equations by cycling through the New Right equations.

   a. $2x-y=4$ and $y=2$

   Answer: $x=3$, $y=2$ and the lines will intersect at (3, 2).

   b. $2x-y=4$ and $x=1$

   Answer: $x=1$, $y=-2$ and the lines will intersect at (1, -2).

   c. $2x-y=4$ and $-x+3y=1$

   Answer: You cannot tell for sure from the graph; the lines will intersect somewhere for x between 2 and 3 and a bit over 1 for y.

   d. $2x-y=4$ and $2x+2y=8$

   Answer: $x=0$, $y=-4$ and the lines will intersect at (0, -4).

Activity 3 [Page 1.7]

1. The equation on the left is $y = 2x + 1$. Answer each of the following and explain how the graph can be used to check your answer.

   a. Select an equation on the right in the form $y = \_\_x + \_\_$ and enter values in the blanks to create a system of equations that will not have a solution. Explain your reasoning.

      Answers will vary. The multiplier of x *as 2 will produce two lines with the same slope.* $y = 2x + a$ *for any value of* a *except 1 will be a line parallel to the given line and produce a system that will not have a solution.*

   b. Select **Edit** and change the equation on the right to one of the form $\_\_x + \_\_y = \_\_$ so the system will have $x = -2$ as the x-coordinate of the point of intersection.

      Answers will vary. Use $x = -2$ *in the first equation to find the value of y that satisfies that equation,* $y = -3$ *and then choose a values for the blanks so that* $(-2, -3)$ *make a true statement, for example,* $5x - 2y = -4$. *Both of the lines should contain the point* $(-2, -3)$.

   c. Select **Edit** and change the equation on the right to one of the form $\_\_x + \_\_y = \_\_$ so that two points that lie on the graph of $y = 2x + 1$ equation are in the solution of the system.

      Answers may vary: –2x + y = 1 would work. There should only be one line in the graph.

2. Reset Page 1.7. Select **New** until you generate a new equation on the left that passes through the vertical axis on the screen. Answer each of the following and explain how the graph can be used to check your answer. Hint: Grabbing and dragging the point on the equation might help you find an answer.

   a. Select an equation on the right in the form $y = \_\_x + \_\_$ and enter values in the blanks to create a system of equations that will not have a solution. Explain your reasoning.

      Answers will vary. The multiplier of x *as 2 will produce two lines with the same slope.* $y = 2x + a$ *for any value of* a *except 1 will be a line parallel to the given line and produce a system that will not have a solution.*

   b. Select **Edit** and change the equation on the right to one of the form $\_\_x + \_\_y = \_\_$ to produce a system of equations with the *x*-intercept of both equations as the solution. (If the point on the *x*-axis for the given line is not visible on the screen, generate a new line.)

      Answers will vary. The point of intersection of the two lines should be on the x*-axis with coordinates of the form (*x*, 0).*

c. Select **Edit** and change the equation on the right to one of the form $\_\_x + \_\_y = \_\_$ to produce a system of equations where one of the equations is a horizontal line.

Answers will vary. One of the equations has to be of the form $y = \_\_$ in order for the solution to lie on a horizontal line.

d. Select **Edit** and change the equation on the right to one of the form $y = \_\_x + \_\_$ to produce an equation whose solution set is a point in the third quadrant.

Answers will vary. Drag the point on the line to a point in the third quadrant. For example, $4x + 5y = 17$ was the equation; drag the point to $(-1, -3)$ in the third quadrant and on the line. Create a new equation such that $(-1, -3)$ makes a true statement, for example $y = 4x + 1$.

3. Identify the following as true or false. Find an example from the TNS activity to support your thinking.

a. If a system does not have a solution, no value of *x* will make both equations in the system true.

b. If two lines intersect at the point (0, 0), the system will not have a solution.

c. If one equation is a multiple of the other, the system will have infinitely many solutions.

d. $x = 3$ and $\boldsymbol{y} = \mathbf{4}$ is a system of equations with the solution (**–3**, 4).

Answer: a) true; b) false; c) true; d) true