Building Concepts: Fractions as Division

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson can be used to develop the relationship between a fraction $\frac{a}{b}$ as ***a*** copies of $\frac{1}{b}$ and the result when the whole number ***a*** is divided by the whole number ***b*** (i.e., the result of dividing 7 whole units into 4 parts can be thought of as 7 copies of $\frac{1}{4}$).

Dividing the whole number ***a*** by another whole number ***b*** can be written as the fraction $\frac{a}{b}$.

Learning Goals

Students should understand and be able to explain each of the following:

1. Dividing a number such as 3 by 4 is the same as three copies of $\frac{1}{4}$;
2. When 3 wholes are shared equally among 4 people, each person has a share of size $\frac{3}{4}$; in general, when ***a*** wholes are shared equally among ***b*** people, each has a share of size $\frac{a}{b}$;
3. Dividing the whole number ***a*** by another whole number ***b*** can be written as the fraction $\frac{a}{b}$;
4. If ***a*** divided by ***b*** equals ***c***, then ***a*** equals the product of ***b*** and ***c.***

Prerequisite Knowledge

Fractions as Division is the eighth lesson in a series of lessons that explore fractions and build on the concepts in the previous lessons: *What is a Fraction? Equivalent Fractions*, and *Create Equivalent Fractions.* Students should be familiar with the terms *unit fraction*, *equivalent fraction*, *common denominator,* and *improper fraction* covered in earlier lessons. Prior to working on this activity students should understand:

- the concept of division
- the concept of equivalent fractions
- how to locate fractions on a number line
- the concept of related multiplication and division facts
- how numbers greater than 1 can be written as improper fractions

Vocabulary

- **divisor:** the number that is dividing into another number; the denominator of the fraction; also called a factor of *n* ,the divisor is an integer that can be multiplied by some other integer to produce *n*
- **dividend:** the number that is to be divided; or the numerator in the fraction

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Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Fractions as Division_Student.pdf
- Fractions as Division_Student.doc
- Fractions as Division.tns
- Fractions as Division_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

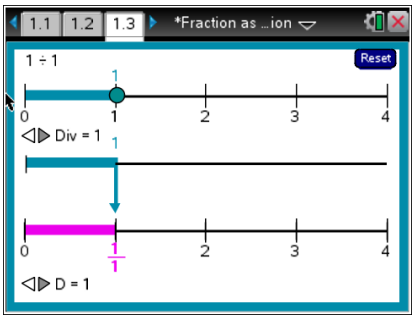
This TI-Nspire™ lesson can be used to develop the relationship between a fraction $\frac{a}{b}$ as ***a*** copies of $\frac{1}{b}$ and the result when the whole number *a* is divided by the whole number *b* (i.e., the result of dividing 7 whole units into 4 parts can be thought of as 7 copies of $\frac{1}{4}$). The file illustrates two different approaches to thinking about this relationship between fractions and division. In the first approach, the whole number ***a*** is partitioned into ***b*** parts, and the value for each of those parts is determined. In the second approach, the whole number ***a*** is partitioned into units; each of those units is partitioned into ***b*** parts, with the value displayed as the sum of the copies of $\frac{1}{b}$ from each of the units.

Note that you get a different mental image from the division problem than from the fraction (i.e., $\frac{7}{4}$ gives you a sense of the size and location of the number where 7 divided into 4 parts is not quite as direct).

Students should recognize that division problems can be expressed as multiplication problems with a missing factor.

Part 1, Page 1.3

Focus: Students will divide a whole number by another whole number to generate a fraction.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

On page 1.3, dragging the dot on the top number line locates a whole number ***a*** on the number line. The arrows above the middle number line set the divisor (Div), which partitions the middle number line into ***b*** equal parts. Choosing D on the bottom number line displays each unit in *a* partitioned into ***b*** intervals with $\frac{1}{b}$ highlighted in each unit. Thus, in dividing 3 by 4, the number 3 on the top number line would show the intervals from 0 to 1, from 1 to 2 and from 2 to 3; each of these partitioned into $\frac{1}{4}$ on the middle number line, and the first $\frac{1}{4}$ on the bottom number line highlighted. Using D to select 4 to partition the bottom number line into $\frac{1}{4}$'s, students can see that the 3 copies of $\frac{1}{4}$ is the same as the whole number 3 divided by 4.

Teacher Tip: Have students suggest whole numbers between 1 and 4 to be divided. Use the interactive number lines in the .tns file to model the division. Encourage students to make predictions about where the dot will be on each number line and what part of the number line would be shaded pink.

As students discuss the concept of fraction as division, encourage them to explain their reasoning. Have students work independently then have the class work together to find the solutions using the interactive number lines.

Class Discussion

Have students…	Look for/Listen for…
Use the file to explain why the result of dividing 4 by 3 is equal to $\frac{4}{3}$.	Possible answer: 4 is divided into three equal parts; this is the same as 4 copies of $\frac{1}{3}$.

Class Discussion (continued)

In each case, use the top and middle number lines on the file to estimate the result of the division problem and check your answer using the bottom number line.

- ***3 divided by 12***

  Answer: $\frac{3}{12}$ or $\frac{1}{4}$.
- ***4 divided by 8***

  Answer: $\frac{4}{8}$ or $\frac{1}{2}$.
- ***4 divided by 10***

  Answer: $\frac{4}{10}$ or $\frac{2}{5}$.
- ***4 divided by 3***

  Answer: $\frac{4}{3}$ or $1\frac{1}{3}$.

Change D on the bottom number line to find at least two equivalent fractions for the answers in the question above.

- Possible answers: 3 divided by 12: $\frac{1}{4}, \frac{2}{8}, \frac{4}{16}, \frac{5}{20}, \ldots$
- Possible answers: 4 divided by 8 : $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{5}{10}, \ldots$
- Possible answers: 4 divided by 10 : $\frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \ldots$
- Possible answers: 4 divided by 3: $\frac{8}{6}, \frac{12}{9}, \frac{20}{15}, \ldots$

Have students…	**Look for/Listen for…**
✓ **Explain the difference between:** (Question #1 on the Student Activity sheet.)	
• ***1 divided by 3 and 2 divided by 6***	Possible answer: 1 divided by 3 is the whole number 1 divided into 3 parts; 2 divided by 6 is the whole number 2 divided into 6 parts. Both $\frac{1}{3}$ and $\frac{2}{6}$ identify the same point on the number line because 1 copy of $\frac{1}{3}$ is the same as 2 copies of $\frac{1}{6}$.

Class Discussion (continued)

- ***4 divided by 5 and 5 divided by 4***

  Possible answer: $\frac{4}{5}$ and $\frac{5}{4}$ are not based on the same unit fraction. $\frac{4}{5}$ is less than 1 and $\frac{5}{4}$ is more than 1.

✓ ***Decide which of the following is correct and explain why:***
Tomas reasoned that if the result of 2 divided by 12 is $\frac{1}{6}$, then the result of

(Question #2 on the Student Activity sheet.)

- ***4 divided by 12 would be twice as much***

  Possible answer: True because you have twice as many whole numbers divided by 12, so you should have twice the results. 2 copies of 2 divided by 12 would be 2 copies of $\frac{1}{6}$, which is $\frac{1}{3}$—twice as much as $\frac{1}{6}$.

- ***4 divided by 12 would be half as much***

  Possible Answer: False because you have more whole numbers to divide by 12 so the answer should be larger, not smaller, than $\frac{1}{6}$.

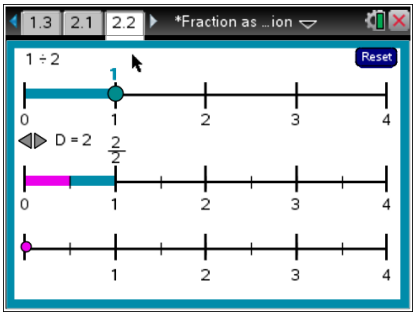
- ***2 divided by 6 would be half as much***

  Possible answer: False. Dividing 2 into only 6 parts would be make a larger amount than dividing 2 into 12 parts, not a smaller.

- ***2 divided by 6 would be twice as much***

  Possible answer: True. Dividing 2 into 6 parts would be 3 parts in each for $\frac{1}{3}$, which is twice as much as $\frac{1}{6}$.

Part 2, Page 2.2



Focus: Students further investigate fractions as a result of dividing whole numbers.

On page 2.2, moving the dot on the top number line locates a whole number ***a*** on the number line. The arrows above the middle number line set the divisor D, which partitions the bottom two number lines into ***b*** equal parts. The middle number line displays each unit in ***a*** partitioned into ***b*** intervals with $\frac{1}{b}$ highlighted in each unit. The dot on the bottom number line can be moved to display the total number of copies of $\frac{1}{b}$ highlighted in the middle number line, i.e. in dividing 3 by 4, the number 3 on the top number line would show the intervals from 0 to 1, from 1 to 2 and from 2 to 3; each of these partitioned into $\frac{1}{4}$'s on the middle number line, with one of the $\frac{1}{4}$'s in each interval highlighted. The dot on the bottom number line can be dragged to show the three $\frac{1}{4}$'s, one for each of the three intervals.

Teacher Tip: Be sure students understand how the interaction with the file supports the mathematics. Asking them how the file is connected to their thinking about the relationship between fractions and division can lead to a productive discussion about the mathematical concept.

Class Discussion

Have students…	Look for/Listen for…
• ***Use the file to explain why the result of dividing 4 by 3 is equal to $\frac{4}{3}$.***	Possible answer: Each of the 4 units in 4 is divided into three parts, and you are looking for one of those parts in each of the 4, which makes the fraction $\frac{4}{3}$.
• ***If 4 people wanted to share 5 pounds of meat, how many pounds would each person get?***	Answer: $\frac{5}{4}$ or $1\frac{1}{4}$ pounds.

Class Discussion (continued)

- ***Sami was figuring out how to share 6 cups of cereal with 4 people and found two answers she thought were different. What do you think these might have been? What would you say to Sami?***

  Possible answer: $\frac{6}{4}$ cups and $\frac{3}{2}$ cups. These are the same amounts but with different denominators: $\frac{6}{4}$ is 1 and two $\frac{1}{4}$ cups and $\frac{3}{2}$ is the same as 1 and $\frac{1}{2}$ cup, but $\frac{1}{2}$ cup is the same as two $\frac{1}{4}$ cups.

✓ ***Answer each of the following. How much would each person get if***
(Question #3 on the Student Activity sheet.)

- ***3 people share 8 pies?*** Answer: $\frac{8}{3}$.
- ***4 people share 8 pies?*** Answer: 2.
- ***3 people share 2 pies?*** Answer: $\frac{2}{3}$.
- ***4 people share 2 pies?*** Answer: $\frac{2}{4}$ or $\frac{1}{2}$.

In general, using either file to reason from or support your answer: the result of 4 divided by 8 is equal to which of the following. Explain why or why not in each case.

- $\frac{2}{6}$ Possible answer: No, subtracting 2 from the numerator and denominator does not produce an equivalent fraction.
- $\frac{3}{6}$ Possible answer: Yes both are equivalent to $\frac{1}{2}$.
- $\frac{2}{4}$ Possible answer: Yes both are equivalent to $\frac{1}{2}$.

Class Discussion (continued)

- ***3/7***

  Possible answer: No, subtracting 1 from the numerator and denominator changes the value of the unit fraction and produces a fraction that cannot be made equivalent to $\frac{1}{2}$.

- ***1/2***

  Possible answer: Yes, it is another name for $\frac{4}{8}$.

- ***The problem*** $10 \div 5 = 2$ ***can be expressed as*** $10 = 5 \times 2$***.***
  Write the division problem $4 \div 9$ ***as a multiplication problem.***

  Answer: $4 = 9 \times \frac{4}{9}$.

- ***Which of the following are equivalent to the statement: 3 divided by 6 is a number y,***

  a. $3 = 6 \times y$ **b.** $3 \div y = 6$ **c.** $6 \div 3 = y$

  d. $y \div 6 = 3$ **e.** $6 = 3 \times y$ **f.** $y = 3 \times 6$

  Answer: a and b.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. If 3 granola bars are to be shared among 4 students how much will each student get?

   a. $\frac{1}{4}$ of a granola bar

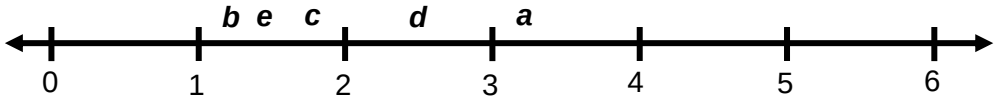
   b. $1\frac{1}{3}$ of a granola bar

   c. $\frac{3}{4}$ of a granola bar

   d. $\frac{1}{2}$ of a granola bar

   Answer: c.

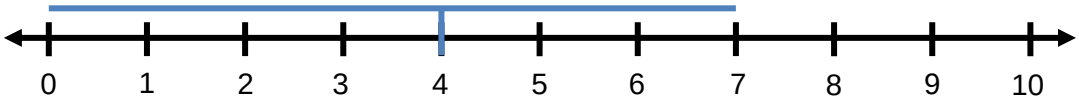
2. Write the letter on the number line to represent the approximate location of each of the following divisions:



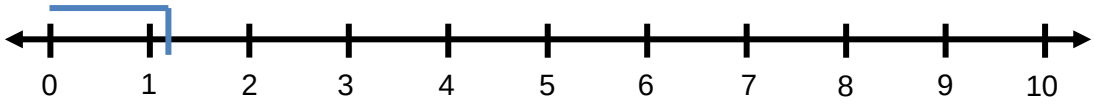
   a. $10 \div 3$ b. $6 \div 5$ c. $5 \div 3$ d. $\frac{10}{4}$ e. $\frac{6}{4}$

3. Which of the following illustrates 7 divided by 4?

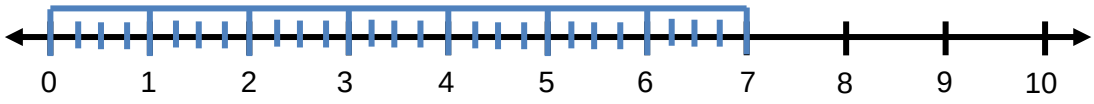
a.



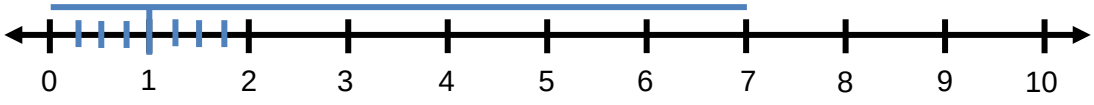
b.



c.



d.



Answer: d.

Student Activity solutions

Vocabulary

divisor: the number that is dividing into another number; the denominator of the fraction; also called a factor of *n* ,the divisor is an integer that can be multiplied by some other integer to produce *n*.

dividend: the number that is to be divided; or the numerator in the fraction.

In this activity, you will divide a whole number by another whole number and write the quotient as a fraction.

1. Explain the difference between

a. 1 divided by 3 and 2 divided by 6

Possible answer: $\frac{1}{3}$ ***and*** $\frac{2}{6}$ ***are the same names for the same number; they are different unit fractions but the same point on the number line.***

b. 4 divided by 5 and 5 divided by 4

Possible answer: $\frac{4}{5}$ ***and*** $\frac{5}{4}$ ***are not based on the same unit fraction.*** $\frac{4}{5}$ ***is less than 1 and*** $\frac{5}{4}$ ***is more than 1.***

2. Decide if the following is correct and explain why.

Tomas reasoned that if the result of 2 divided by 12 is $\frac{1}{6}$, then the result of 4 divided by 12 would be twice as much.

Possible answer: True because you have twice as many whole numbers divided by 12, so you should have twice the results. 2 copies of 2 divided by 12 would be 2 copies of $\frac{1}{6}$***, which is*** $\frac{1}{3}$ ***- twice as much as*** $\frac{1}{6}$***..***

3. Answer each of the following. How much would each person get if

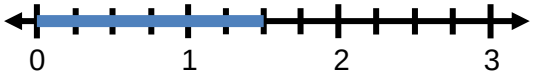
a. 3 people equally share 8 pies? ***Answer:*** $\frac{8}{3}$.

b. 4 people equally share 8 pies? ***Answer: 2***.

c. 3 people equally share 2 pies? ***Answer:*** $\frac{2}{3}$.

d. 4 people equally share 2 pies? ***Answer:*** $\frac{2}{4}$ ***or*** $\frac{1}{2}$.

4. If 6 art students wanted to equally share 4 pounds of clay, how many pounds of clay would each student receive? Complete the number line to solve the problem. Then, write a division and multiplication sentence to show your reasoning.



Answer: $\frac{6}{4}$ **or** $1\frac{1}{2}$ ***pounds*****;** $6 \div 4 = 1\frac{1}{2}$ ***and*** $4 \times 1\frac{1}{2} = 6$.