Building Concepts: Connecting Ratios and Scaling

TEACHER NOTES

Lesson Overview

In this TI-Nspire™ lesson, students investigate ratios and scale factors. Scale factors are ratios that can be used to make a figure smaller or larger, depending on whether the scale factor is smaller than or larger than 1. When a figure is scaled, the ratios between the corresponding sides of the two figures are equivalent, and the internal ratios between two sides of one figure are equivalent to the internal ratio for the corresponding sides of the second figure.

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Learning Goals

1. Write an equation to describe how sides of a figure are related by a scale factor;
2. identify the scale factor given that one figure has been scaled from another;
3. solve problems involving missing dimensions knowing one figure has been scaled from another.

Prerequisite Knowledge

Ratios and Scaling is the fourteenth lesson in a series of lessons that investigates ratios and proportional relationships. In this lesson, students use ratios to solve scaling problems. Prior to working on this lesson, students should have completed *Connecting Ratios to Equations, Proportional Relationships, Solving Proportions,* and *Adding Ratios*. Students should:

- be able to write equations to describe proportional relationships.

Vocabulary

- **scale factor**: the ratio formed by the lengths of the corresponding sides of two figures when one is a scaled copy of the other. This ratio might be expressed as the value of the ratio, the fractional value associated with the ratio.

Lesson Pacing

This lesson should take 50 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Connecting Ratios and Scaling_Student.pdf
- Connecting Ratios and Scaling_Student.doc
- Connecting Ratios and Scaling.tns
- Connecting Ratios and Scaling_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In this TI-Nspire™ lesson, students investigate ratios and scale factors. Ratios are connected to geometry in several ways. One figure can be scaled from another by using a scale factor, making the figure larger or smaller depending on whether the factor is larger or smaller than 1. If the scale factor is k, then each linear measure x will become kx. Another way to reason about scale factors is to think of proportional relationships: every linear measurement x will become $y = kx$, where the constant of proportionality is now the scale factor. When a figure is scaled, the ratios between the corresponding sides of the two figures are equivalent, and the internal ratios between two sides of one figure are equivalent to the internal ratio for the corresponding sides of the second figure. Knowing that one figure is scaled from another, students can solve problems involving proportions to find missing dimensions.

Note that scaling can be associated with similar figures, but this lesson is an informal introduction to the ideas related to similarity and the language of similarity is not introduced here.

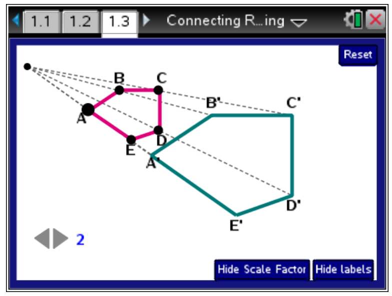
Part 1, Page 1.3

Focus: What is a scale factor and how can scale factors help you solve problems involving two scaled shapes?

On page 1.3, the scale factor is in the lower left corner of the screen and can be changed by using the arrow keys.

Move each of the points in the figure to change its shape. Move the point of dilation on the screen to change the perspective. Select the sides of the shape to show side lengths.

On the handheld, you can also hide/show the scale factor and labels by selecting the menu key.



TI-Nspire Technology Tips

Use the tab key to toggle between the points in the original figure and the scale factor arrows

Reset returns to the original screen or press ctrl del on handheld to reset.

Class Discussion

Teacher Tip: This question introduces students to the notion of corresponding sides between two scaled figures and to the concept of a scale factor that relates the sides of one figure to the sides of the other.

Figure A'B'C'D'E' has been scaled from figure ABCDE.

- ***Which side of the new figure corresponds to side AB? to side CD?***

  Answer: A′B′ corresponds to AB, and CD corresponds to C′D′.

- ***The scale factor is given. Select the sides of AB and C'D'. Predict what you think sides A'B' and CD will be. Then select those sides to check your prediction.***

  Answer: A′B′ will be 6, and CD will be 4.

Class Discussion (continued)

> **Teacher Tip:** In the following questions, students use the relationship between the sides of two scaled figures and the scale factor to find missing dimensions in the scaled figure and to solve problems involving scale factors.

Have students…	*Look for/Listen for…*
Find the scale factor that will make the given ratios between the sides of A′ B′ C′ D′ E′ correspond to the sides of ABCDE. Give an example using a side from each figure to support your answer.	
• ***2 to 1***	Answer: a scale factor of 2 because when *CD* is 4, *C′D′* is 8; *C′D′*: *CD* will be 8:4, which is equivalent to 2:1.
• ***1 to 2***	Answer: a scale factor of $\frac{1}{2}$ because when *CD* is 4, *C′D′* will be 2; *C′D′*: *CD* will be 2:4, which is equivalent to 1:2.
• ***1 to 1***	Answer: a scale factor of 1 because when *CD* is 4, *C′D′* is four and the ratio is 4:4, which is equivalent to 1:1.
Reset the page, and then hide the scale factor. Select the left arrow.	
• ***Select a side of the larger figure and the corresponding side of the smaller figure. What is the scale factor? Justify your thinking.***	Answer: I selected *AE* = 5 and *A′E′* = 10. Thus, the scale factor is $\frac{A'E'}{AE}$ or $\frac{10}{5}=\frac{2}{1}$ because the side of the new figure is twice the length of the side of the original figure.
• ***Select all of the sides of ABCDE to find their measures and use the scale factor to find the corresponding measurements in the scaled figure. Check your answer using the TNS lesson.***	Answer: $A`B`=6$, $B`C`=8$, $C`D`=8$, $D`E`=4$, and $E`A`=10$

Student Activity Questions—Activity

1. **What will each of the following scale factors do to ratios of each side length of *ABCDE* to the corresponding side length of *A'B'C'D'E'*?**

   a. **scale factor of 1**

   Answer: *A'B'C'D'E'* will be the same as *ABCDE*. The figure will not change.

   b. **scale factor of $\frac{1}{2}$**

   Answer: Each segment in *A'B'C'D'E'* will be half as long as the corresponding segment in *ABCDE*.

   c. **scale factor of 4**

   Answer: Each segment in *A'B'C'D'E'* will be four times as long as the corresponding segment in *ABCDE*.

2. **Reset the page. Be sure the scale factor is hidden. Select the right arrow and change the scale factor once. Find *AB* and *A'B'*.**

   a. **How can you use this information to find the scale factor?**

   Answer: *A'B'* is scaled up from 3 to $\frac{15}{2}$, so the scale factor is $\frac{5}{2}$.

   b. **Find the lengths of *A'E'* and *E'D'*. Explain your reasoning.**

   Answer: *A'E'* will be $\frac{25}{2}$ because *AE* is 5. Since $\frac{5}{2}$ times 5 will be $10+\frac{5}{2}$ or 312.5, *ED* is 2; so *E'D'* will be $\frac{5}{2}$ times 2 or 5.

3. **If you know one length in figure 1 is L1 and the corresponding length in figure *A'B'C'D'E'* is L2, which of the following will give the scale factor between *ABCDE* and *A'B'C'D'E'*? To go from L1 to L2? Explain your reasoning and give an example that supports your claim using the TNS lesson.**

   a. $\frac{\mathbf{L1}}{\mathbf{L2}}$ **b.** $\frac{\mathbf{L2}}{\mathbf{L1}}$ **c.** $\mathbf{L1+L2}$ **d.** $\frac{(\mathbf{L2-L1})}{\mathbf{L1}}$

   Answer: b. Examples will vary; one possible example is when $BC=4$ and $B`C`=8$, the scale factor is choice b, $\frac{8}{4}=\frac{2}{1}$ or 2, not any of the others.

Student Activity Questions—Activity (continued)

4. Reset the page, and then hide the scale factor. Change the scale factor using the right arrow three times so that *D'E'* is off the screen. Reveal the length of segments *AB* and *A′B′*. Find the length of hidden side *D'E'* using at least two different strategies.

Possible answer: One strategy is to find the scale factor from *AB* (3) and *A′B′* (10.5) by setting up the ratio 10.5 to 3, which is equivalent to 3.5 to 1 so the scale factor is 3.5. Use the scale factor and the equation y = 3.5x to find the other sides. $D'E' = 3.5 \times 2 = 7$. A second strategy is to use the fractions associated with the ratios, $\frac{AB}{DE} = \frac{A'B'}{D'E'}$; $\frac{3}{2} = \frac{10.5}{D'E'}$; multiply the numerator and denominator in the first fraction by 3.5 to get $\frac{10.5}{7} = \frac{10.5}{D'E'}$, so *D′E′* would be 7.

Additional Discussion

Have students…	Look for/Listen for…
Move the points in figure ABCDE to create as many segments that are the same length as you can.	
If the scale factor is 2.5,	
• ***what equation can be used to find the lengths of the segments in A'B'C'D'E'?***	Answer: $y = 2.5x$, where y is one of the segments in A′B′C′D′E′ and x is one of the segments in ABCDE.
• ***what will the lengths of the segments in A'B'C'D'E' be? Explain how you found your answers.***	Answers are not unique. One response could be: If three sides are the same length, 7, $A`B` = 17.5$, $B`C` = 17.5$, $C`D` = 7.5$, $D`E` = 17.5$, and $E`A` = 15$,. Some of the answers you can find by selecting the sides of *A′B′C′D′E′*. But for all of them, you can use the equation from part a with the five sides in *ABCDE* as the set of *x*-values.
Move the point in the upper left corner so that it is in the "center" of ABCDE.	
• ***What do you observe about the relationship between the two figures?***	Possible answers: They are the same shape; the angles seem to be the same.
• ***Set the scale factor to 2.5. Select A'E'. Use that information to find the length of AE. Check your answer using the TNS lesson.***	Possible answer: $A'E' = 12.5$. Using the equation $A'E' = 2.5AE$ gives $12.5 = 2.5AE$. Dividing both sides by 2.5 produces *AE* = 5.

Additional Discussion (continued)

Have students	Look for/Listen for…
Move points A and C. Set the scale factor to 2.5. An internal ratio is a ratio between two sides of the same figure. Identify which of the following proportional relationships represent internal ratios and which represent ratios between the figures? Explain your reasoning in each case.	
• $\frac{6}{5} = \frac{15}{12.5}$	Answer: An internal ratio because each fraction comes from one figure; *AB*/*AE* corresponds to $\frac{A'B'}{A'E'}$. (Reasons may vary.)
• $\frac{5}{7} = \frac{15}{17.5}$	Answer: Not possible because the segments are not corresponding parts. (Reasons may vary.)
• $\frac{2}{5} = \frac{7}{17.5}$	Answer: Between figures because the numerators of the fractions come from *ABCDE* and the denominators of the fractions from *A'B'C'D'E'*; $\frac{ED}{E'D'}$ corresponds to $\frac{BC}{B'C'}$. (Reasons may vary.)
• $\frac{15}{6} = \frac{5}{2}$	Answer: Between figures because the numerators of the fractions from *A'B'C'D'E'* and the denominators of the fractions come from *ABCDE*; $\frac{A'B'}{AB} = \frac{E'D'}{ED}$. (Reasons may vary.)

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. The length of a photograph is 5 inches and its width is 3 inches. The photograph is enlarged proportionally. The length of the enlarged photograph is 10 inches. What is the width of the enlarged photograph?

    a. 6 inches

    b. 7 inches

    c. 9 inches

    d. 15 inches

    e. $16\frac{2}{3}$ inches

    NAEP 2013, grade 8

    Answer: a) 6 inches

2. Dan created a map of the state of Ohio using a scale factor of 2 centimeters: 25 miles. The actual distance from Cleveland to Columbus is 142 miles. About how far apart are Cleveland and Columbus on Dan's map?

    a. 5.70 cm

    b. 11.40 cm

    c. 71.00 cm

    d. 1,775.00 cm

    Ohio Achievement Test 2008, Grade 7

    Answer: b) 11.40 cm

3. Lisa's father is an architect. He builds a cardboard model of each building he designs. The scale of his model to the actual building is 1 inch = 8 feet. How tall will the actual building be when the model is 36 inches tall?

    a. 24 feet

    b. 96 feet

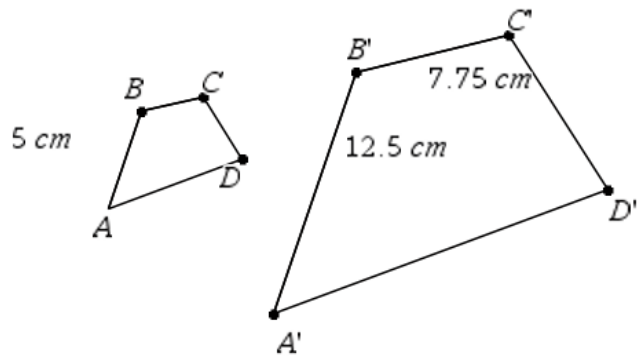
    c. 192 feet

    d. 288 feet

    adapted from Ohio Achievement Test 2008, grade 7

    Answer: d) 288 feet

4. Figure *A′B′C′D′* has been scaled from figure *ABCD*.



   a. What is the scale factor? ***Answer: 2.5***

   b. What is length *BC*? ***Answer: 3.1***

Student Activity Solutions

In this activity you will use scale factors to solve problems involving two scaled shapes. After completing the activity, discuss and/or present your findings to the rest of the class.

Activity [Page 1.3]

1. What will each of the following scale factors do to ratios of each side length of ABCDE to the corresponding side length of *A'B'C'D'E'*?

   a. scale factor of 1

   Answer: A'B'C'D'E' will be the same as ABCDE. The figure will not change.

   b. scale factor of $\frac{1}{2}$

   Answer: Each segment in A'B'C'D'E' will be half as long as the corresponding segment in ABCDE.

   c. scale factor of 4

   Answer: Each segment in A'B'C'D'E' will be four times as long as the corresponding segment in ABCDE.

2. Reset the page. Be sure the scale factor is hidden. Select the right arrow and change the scale factor once. Find AB and *A'B'*.

   a. How can you use this information to find the scale factor?

   Answer: A'B' is scaled up from 3 to $\frac{15}{2}$*, so the scale factor is* $7\frac{1}{2}$.

   b. Find the lengths of *A'E'* and *E'D'*. Explain your reasoning.

   Answer: A'E' will be 37.5 because AE is 5. Since $7\frac{1}{2}$ *times 5 will be* $35+\frac{5}{2}$ *or 37.5, ED is 2; so E'D' will be* $\frac{15}{2}$ *times 2 or 15.*

3. If you know one length in figure 1 is L1 and the corresponding length in figure *A'B'C'D'E'* is L2, which of the following will give the scale factor between *ABCDE* and *A'B'C'D'E'*? To go from L1 to L2? Explain your reasoning and give an example that supports your claim using the TNS lesson.

   a. $\frac{\text{L1}}{\text{L2}}$ b. $\frac{\text{L2}}{\text{L1}}$ c. $\text{L1}+\text{L2}$ d. $\frac{(\text{L2}-\text{L1})}{\text{L1}}$

   Answer: b. Examples will vary; one possible example is when $BC=4$ *and* $B`C`=8$*, the scale factor is choice b,* $\frac{8}{4}=\frac{2}{1}$ *or 2, not any of the others.*

4. Reset the page, and then hide the scale factor. Change the scale factor using the right arrow three times so that *D′E′* is off the screen. Reveal the length of segments *AB* and *A′B′*. Find the length of hidden side *D′E′* using at least two different strategies.

   Possible answer: One strategy is to find the scale factor from AB (3) and A'B' (10.5) by setting up the ratio 10.5 to 3, which is equivalent to 3.5 to 1 so the scale factor is 3.5. Use the scale factor and the equation y = 3.5x to find the other sides. $D'E' = 3.5 \times 2 = 7$. *A second strategy is to use the fractions associated with the ratios,* $\frac{AB}{DE} = \frac{A'B'}{D'E'}$; $\frac{3}{2} = \frac{10.5}{D'E'}$; *multiply the numerator and denominator in the first fraction by 3.5 to get* $\frac{10.5}{7} = \frac{10.5}{D'E'}$, *so D'E' would be 7.*