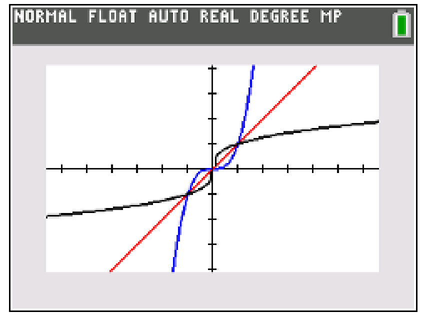
Inverse Functions
Student Activity

Name ________________
Class ________________

The goal of this activity is to help students understand that an inverse function, $f^{-1}(x)$, reverses or undoes the effect of a function, is a reflection in the line *y* = *x*, and that the domain of $f^{-1}(x)$ is equal to the range of $f(x)$. Students will ultimately try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.



The prior knowledge needed for this activity is the relationship between a function and its inverse. The first few problems will revisit these relationships before asking you to apply this knowledge to real world scenarios.

First, we will review the graphical relationship. Graph the function $f(x) = -1 + \sqrt{x-1}$ and the line $y = x$ into Y1 and Y2 respectively.

Problem 1

Take a moment to discuss with a partner the significance of the line $y = x$ with respect to the function and its inverse. Share your thoughts with the class.

Problem 2

Now let us discuss an algebraic relationship a function has with its inverse, finding a function's inverse. We use a two-step process. With the given function, you will first switch the x and y, and second, you will solve for y:

Given Function:

$y = -1 + \sqrt{x-1}$

Switch x and y:

$x = -1 + \sqrt{y-1}$

Solve for y:

$x + 1 = \sqrt{y-1}$

$(x+1)^2 = y - 1$

$y = f^{-1}(x) = (x+1)^2 + 1$

**Further practice:

Find the inverse of each function.

(a) $f(x) = 3x - 7$

(b) $f(x) = \sqrt[3]{x+5} - 2$

(c) $f(x) = 2 + \frac{5}{x-4}$

Problem 3

Graph the example demonstrated from **Problem 2** into Y3. Discuss with a partner what you notice after graphing the new function. Share your thoughts with the class.

Problem 4

Extension Question:

What is the relationship between the domain and the range of a function and its inverse?

Use the graphical and algebraic relationships on the previous pages to discuss this with a partner.

Share your results with the class.

Problem 5

Real World Inverse Function Applications Example 1:

Temperature Conversions

(°F → °C and °C → °F)

The formula to convert temperatures from degrees Celsius to Fahrenheit is $°\mathrm{F} = \frac{9}{5} \cdot °\mathrm{C} + 32$.

(a) Write the inverse function, which converts temperatures from Fahrenheit to Celsius.

(b) Find the Celsius temperature that is equal to 89 degrees Fahrenheit.

(c) Explain how you could have found the answer to part (b) without finding the inverse function.

Problem 6

Real World Inverse Function Applications Example 2:

Money Conversions

A Canadian traveler who is heading to the United States exchanges some Canadian dollars for U.S. dollars. At the time of his travel, $1 Can = $0.79 U.S.
At the same time an American business woman who is in Canada is exchanging some U.S. dollars for Canadian dollars at the same exchange rate.

(a) Find the amount of money in U.S dollars that the Canadian traveler would get if he exchanged $500.

(b) Write an equation that gives the amount of money in U.S. dollars, d, as a function of the Canadian dollar amount, c, being exchanged.

(c) Find the amount of money in Canadian dollars that the American Business woman would get if she exchanged $1000 U.S.

(d) Explain why it might be helpful to write the inverse of the function you wrote in part (b) to answer part (c). Then, write an equation that defines the inverse function.

Further IB style question:

The price of a liter of soda at Carl's Convenient Store is $1.20. Carl's is having a sale on soda. If you purchase a minimum of 8 liters, a $4 discount is applied to your total. This can be modeled by the function, S, which gives the total cost when buying a minimum of 8 liters of soda.

$$S(x) = 1.20x - 4, \qquad x \geq 8$$

(a) Find the total cost of buying 10 liters of soda at Carl's. [2 marks]

(b) Find $S^{-1}(26)$. [2 marks]