2017 AP Calculus Exam: AB-3/BC-3 Technology Uses and Problem Extensions

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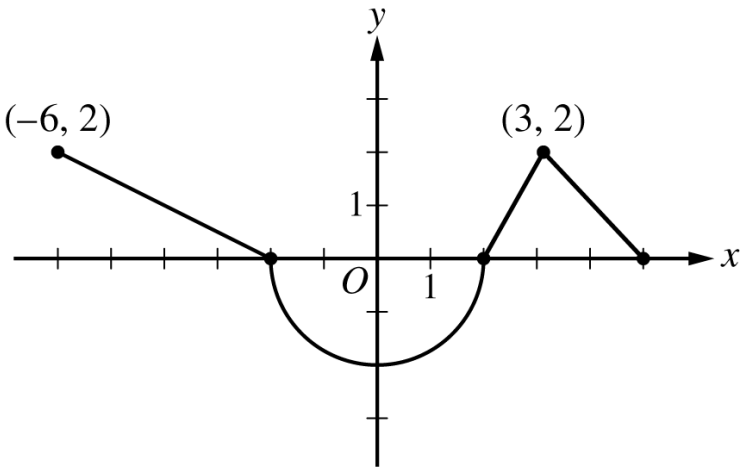
AB-3/BC-3



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is f increasing? Justify your answer.

   (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi$

$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.
Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$f'(x) = 0 \Rightarrow x = -2, x = 2$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$

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(d) $f''(-5) = \dfrac{2-0}{-6-(-2)} = -\dfrac{1}{2}$

$$\lim_{x\to 3^-} \frac{f'(x)-f'(3)}{x-3} = 2 \text{ and } \lim_{x\to 3^+} \frac{f'(x)-f'(3)}{x-3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x\to 3^-} \frac{f'(x)-f'(3)}{x-3} \neq \lim_{x\to 3^+} \frac{f'(x)-f'(3)}{x-3}.$$

$$2 : \begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \quad \text{with explanation} \end{cases}$$

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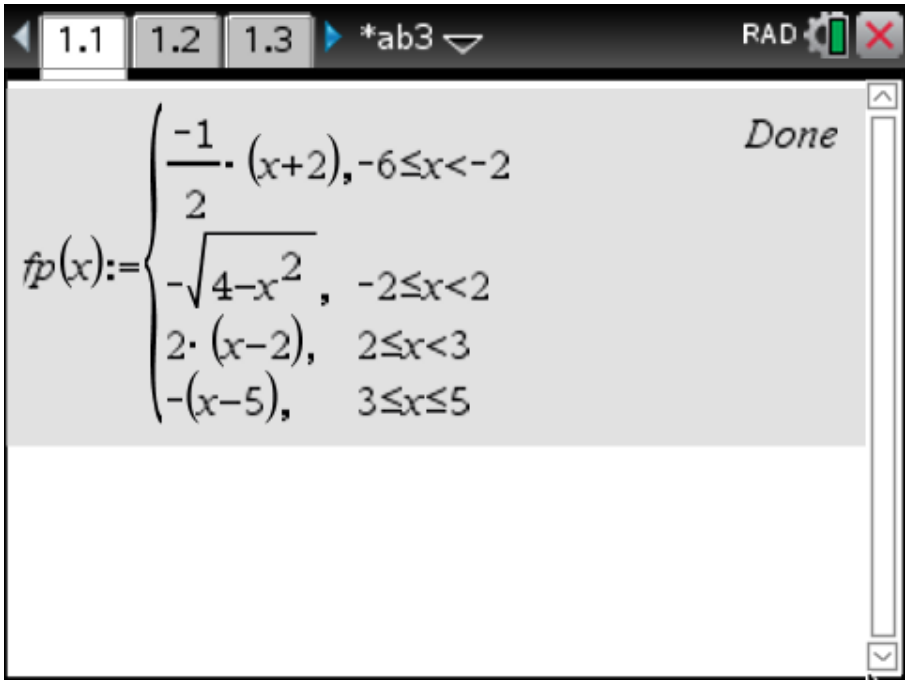
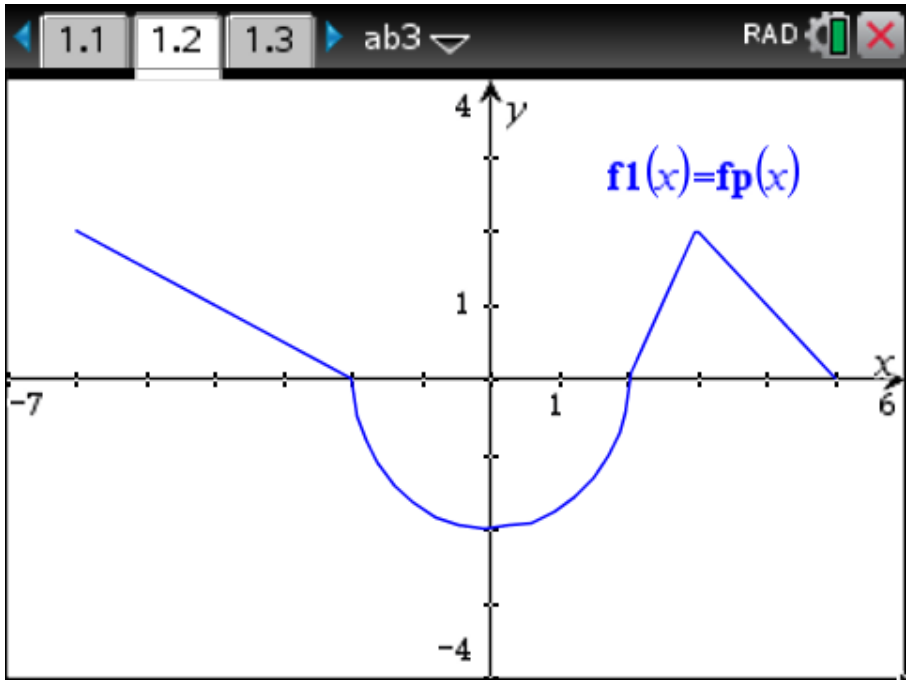
Part (a)

Technology Solution

(1) Determine the equation of each part of the curve.

(2) Define a piecewise function.





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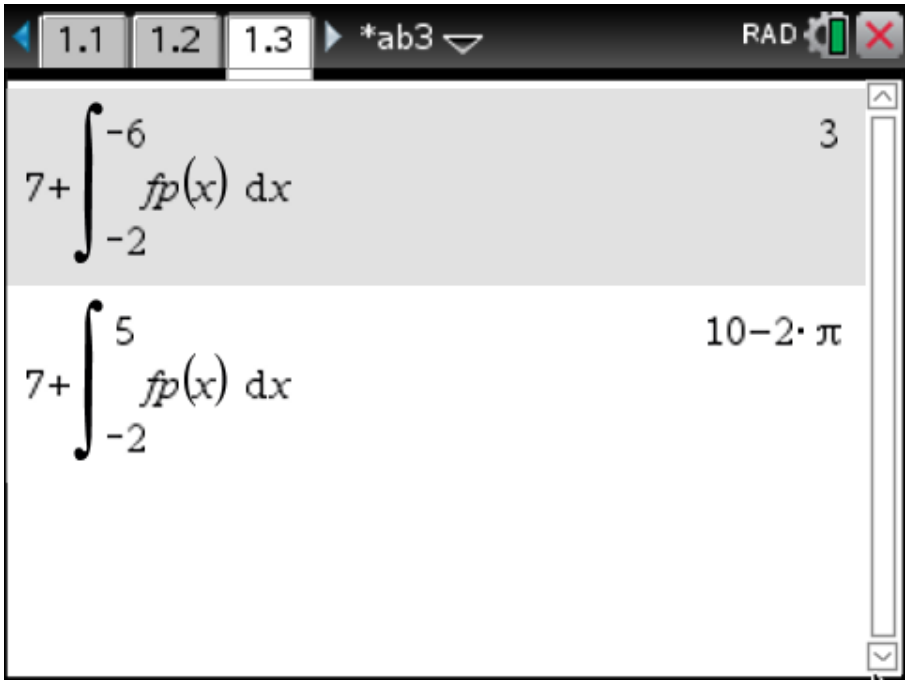
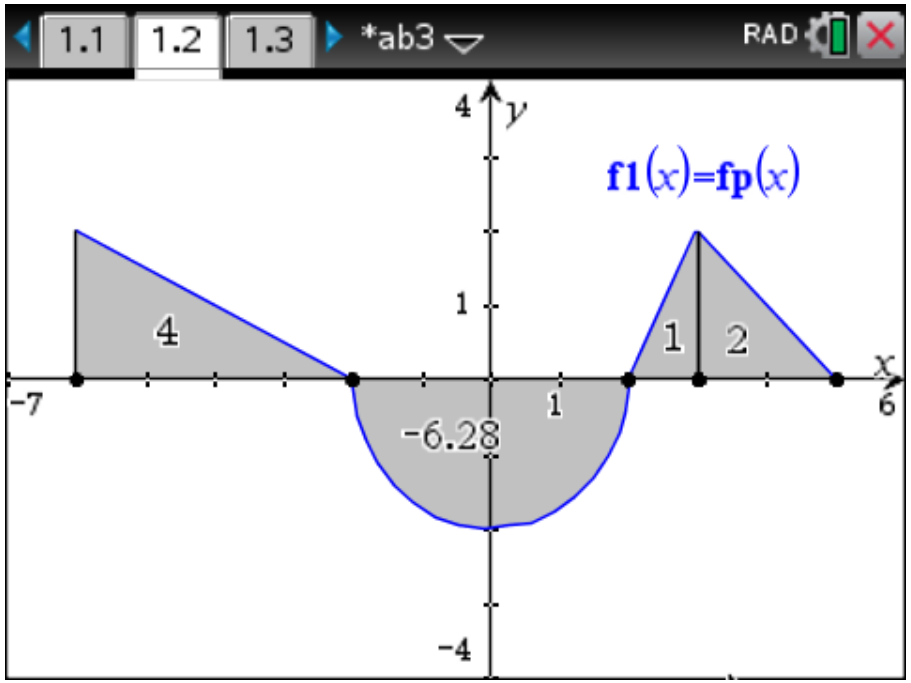
Part (a)

Technology Solution

(1) Find the values $f(-6)$ and $f(5)$ analytically.

(2) Find the area of each region numerically (on a graph screen).





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Part (b) Problem Extensions

(1) On what intervals is f decreasing? Justify your answer.

Solution

$f'(x) < 0$ on the interval $(-2, 2)$.

Therefore, f is increasing on the interval $[-2, 2]$.

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Part (b) Problem Extensions

(2) On what open intervals is the graph of f concave up? Concave down? Find the point(s) of inflection.

Solution

The graph of f is concave up on the interval $(0, 3)$ because f' is increasing on this interval.

The graph of f is concave down on the intervals $(-6, 0)$ and $(3, 5)$ because f' is decreasing on these intervals.

The graph of f has a point of inflection at $x = 0$ because f' changes from decreasing to increasing at this point.

The graph of f has a point of inflection at $x = 3$ because f' changes from increasing to decreasing at this point.

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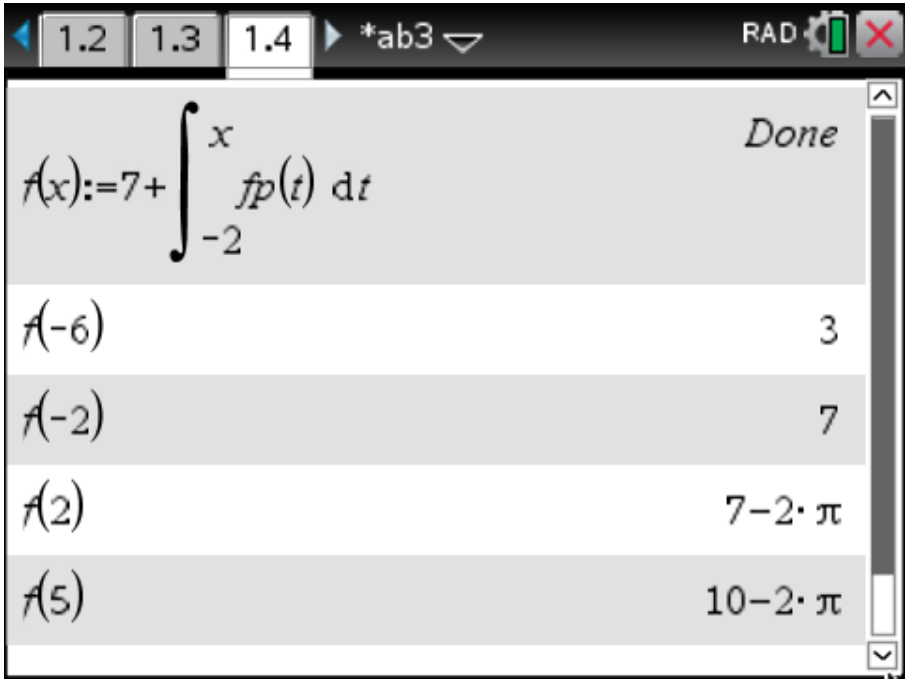
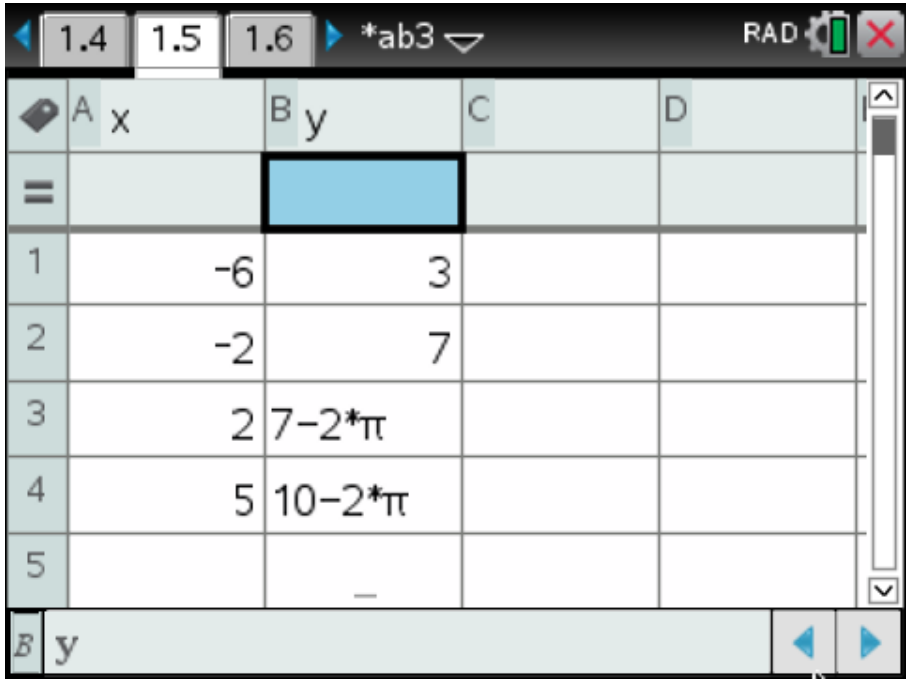
Part (c)

Technology Solution

Define the function f. Evaluate f at the critical pints and endpoints of the interval.





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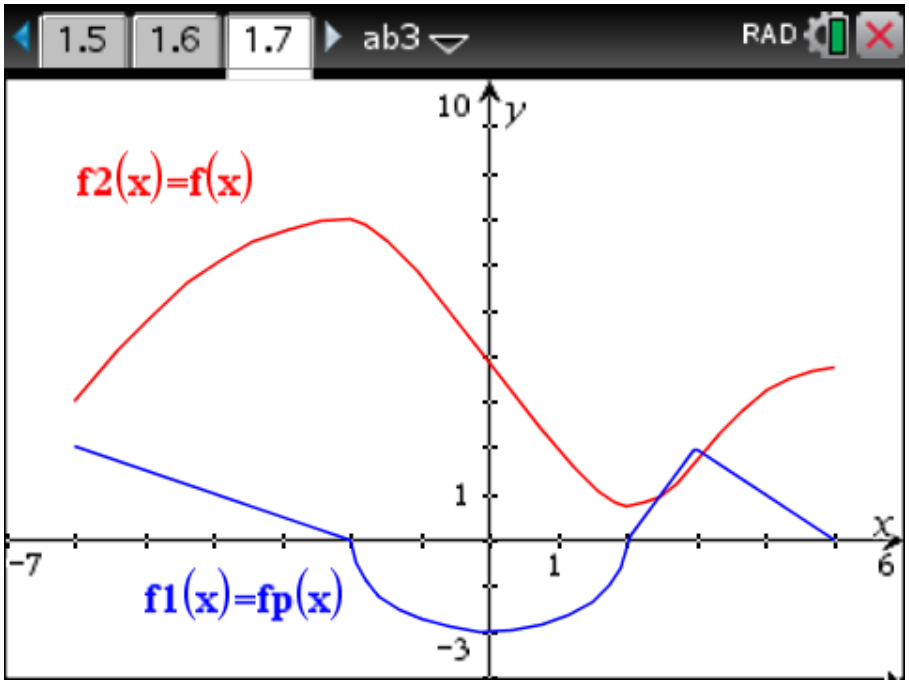
Part (c) Problem Extensions

Carefully sketch a graph of $y = f(x)$.



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Part (c) Problem Extensions

For each function find:

(a) The domain and range.

(b) The intervals on which the graph of the function is increasing, decreasing.

(c) The intervals on which the graph of the function is concave up, concave down.

(d) Find the inflection point(s) on the graph of the function.

(e) Find the absolute extreme values for the function.

(1) $g(x) = \int_0^{\sqrt{x}} f'(t)\,dt$

(2) $h(x) = 2x + \int_0^{x} f'(t)\,dt$

(3) $j(x) = e^{-x^2} + \int_0^{x} f'(t)\,dt$

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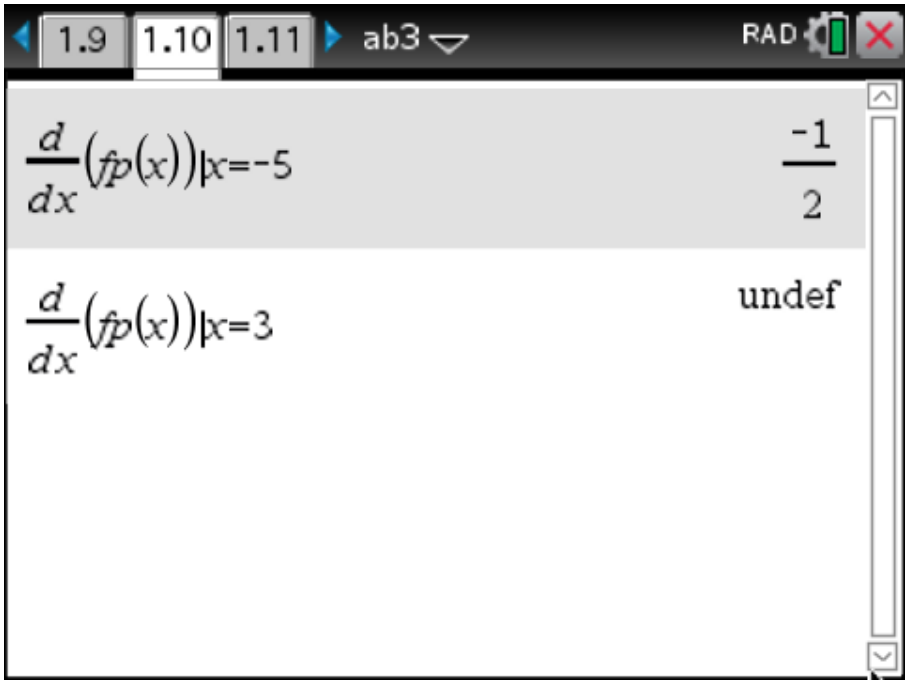
Part (d)

Technology Solution

Find $f''(-5)$ and $f''(3)$.



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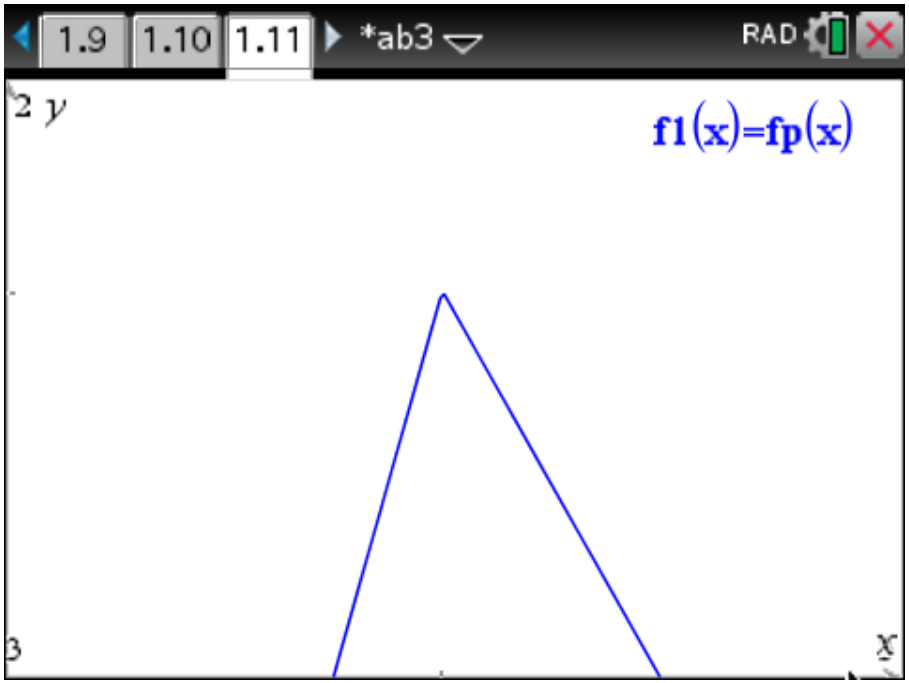
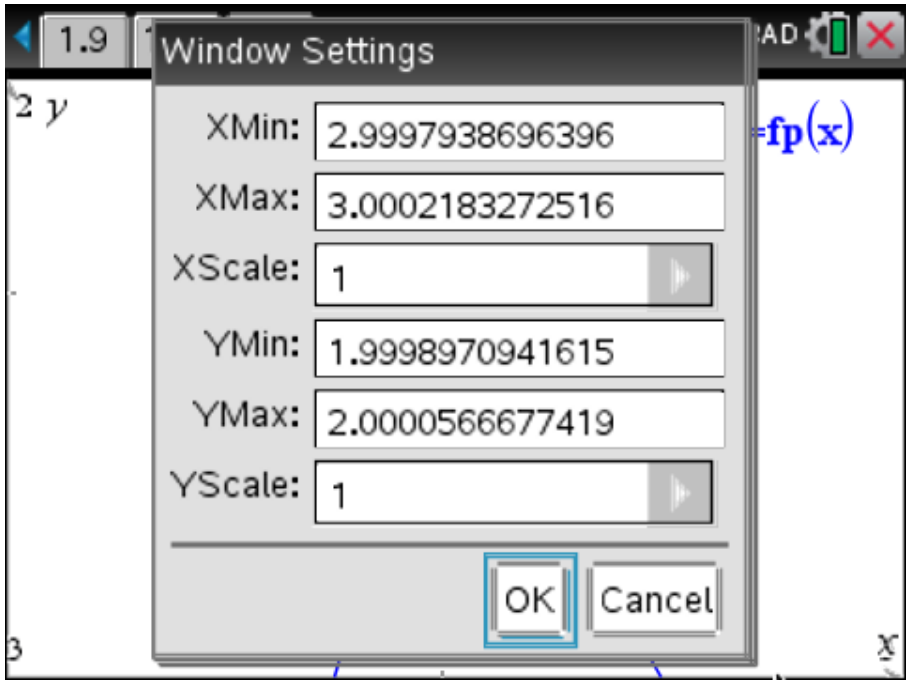
Part (d)

Technology Solution(Graphically)

If the function f' is differentiable on an interval containing a, then f' can be made to appear locally linear near a.

Zoom in near the point $(3, 2)$.